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Real-time Decentralized Voltage Control in Distribution Networks

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Abstract— **Voltage control plays an important role in the operation of electricity distribution networks, especially when there is a large penetration of renewable energy resources. In this paper, we focus on voltage control through reactive power compensation and study how different information structures affect the control performance. In particular, we first show that only using voltage measurements to determine reactive power compensation is insufficient to maintain voltage in the acceptable range. Then we proposes two fully decentralized algorithms by slightly adding additional information into the control design. The two algorithms are guaranteed to stabilize the voltage in the acceptable range regardless of the system operating condition. The one with higher complexity can further minimize a cost of reactive power compensation in a particular form. Both of the two algorithms use only local measurements and local variables and require no communication.**

I. Introduction

Voltages in a distribution feeder fluctuate according to the feeder loading condition. The primary purpose of voltage control is to maintain acceptable voltages (plus or minus 5% around nominal values) at all buses along the distribution feeder under all possible operating conditions. Traditionally the voltage control is achieved by re-configuring transformer taps and capacitors banks (Volt/Var control) [1], [2] based on local measurements (usually voltages) at a slow time scale. This control setting works well under normal circumstances, because the change of the loading condition is relatively mild and predictable.

Due to the increasing penetration of distributed energy resources (DER) such as photovoltaic and electric vehicles in the distribution networks, the operating conditions (supply, demand, voltages, etc) of the distribution feeder fluctuate fast and by a large amount. The conventional voltage control lacks flexibility to respond to those conditions and they may not produce the desired results. This raises important issues on the network security and reliability. To overcome the challenges, new technologies are being proposed and developed, e.g., the inverter design for voltage control. Inverters connect DERs to the grid and adjust the reactive power outputs to stabilize the voltages at a fast time scale [3], [4]. The new technologies will enable realtime distributed voltage control that is needed for the future power grid.

One key element to implement those new technologies is the voltage control rules which satisfy certain information constraints yet guarantee the overall system performance. In general, in the low/medium voltage distribution networks, only a small portion of buses are monitored, individuals are unlikely to announce their generation or load profile, and the availability of DERs are fluctuating and uncertain. All of these facts demand decentralized algorithms for the voltage control. Each control component adjusts its reactive power input based on the local signals that are easy to measure, to calculate, or to communicate. The local information dependence facilitates the realtime implementation of those algorithms. In fact, there exist classes of inverter-based local voltage control schemes that only use local voltage measurements [5], [6]. However, it remains as a daunting challenge to guarantee the performance of the control rules, i.e., to stabilize the voltages within the acceptable range under all possible operating conditions.

In this paper, we focus on voltage control through reactive power compensation.[1] To facilitate the design and analysis of voltage control, we use a linear branch flow model similar to the Simplified DistFlow equations introduced in [7]. The linear branch flow model and the local Volt/Var control form a closed loop quasi-dynamical system (Equation (4)). Then we study three types of voltage control with different information structure. In particular, we first show that using only voltage measurements to determine reactive power compensation is insufficient, even if the controller use the global voltage information. Then we proposes two decentralized algorithms by adding additional information into the control design. The additional information can be either measured locally or computed locally, meaning that the two algorithms are fully decentralized, requiring ***no communication***. With the aid of the additional information, both of the two algorithms can stabilize voltage in the acceptable range; and the one with higher complexity can further minimize the cost of the reactive power compensation in a particular form. Lastly, we complement our analysis through numerical case studies. Though the theoretical analysis is built on the linearized branch flow model, the numerical studies use the full nonlinear AC power flow model, demonstrating the applicability of our approaches in the real systems. Our results implies that with the aid of right local information, local voltages carry out the whole network information for Volt/Var control.

Research has been proposed and conducted to improve the existing voltage control to mitigate the voltage fluctuation impact. To name a few, [8] studies the active power

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[1]Thus, the two terms, voltage control and Volt/Var control, are interchangeable in this paper.

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curtailment to mitigate the voltage rise impact caused by DER; [9] studies the distributed VAR control to minimize power losses and stablizie voltages; [10] reverse-engineers the IEEE 1547.8 standard and study the equilibrium and dynamics of voltage control; [11] proposes two stage voltage control. Compared with the work in this paper, they usually require certain amount of communication or lack theoretical guarantee of performance.

The remaining of this paper is organized as follows: Section II present an AC power flow model, its linear approximation, and the formulation of the Volt/Var control; Section II-C illustrate the impossibility result of merely using voltage information in the control; Section IV presents one decentralized algorithm to maintain acceptable voltages; Section V presents one decentralized algorithm to maintain acceptable voltages and also reach certain optimality as to the reactive power support; Section VI simulate the algorithms to complement our analysis.

II. Preliminaries: Power Flow Model and Problem Formulation

Due to space limit, we introduce here an abridged version of the branch flow model; see, e.g., [12], [13] for more details.

A. Branch flow model for radial networks

Consider a radial distribution circuit that consists of a set N of buses and a set E of distribution lines connecting these buses. We index the buses in N by $i = 0, 1, \ldots, n$, and denote a line in E by the pair (i, j) of buses it connects. Bus 0 represents the substation and other buses in N represent branch buses. For each line $(i, j) \in E$, let $I_{i,j}$ be the complex current flowing from buses i to j, $z_{ij} = r_{ij} + \mathbf{i}x_{i,j}$ be the impedance on line (i, j), and $S_{ij} = P_{ij} + \mathbf{i}Q_{i,j}$ be the complex power flowing from buses i to bus j. On each bus $i \in N$, let V_i be the complex voltage and $s_i = p_i + \mathbf{i}q_i$ be the complex power injection, i.e., the generation minus consumption. As customary, we assume that the complex voltage V_0 on the substation bus is given and fixed at the nominal value.

The branch flow model was first proposed in [1], [2] to model power flows in a steady state in a radial distribution circuit:

$$-p_j = P_{ij} - r_{ij}\ell_{ij} - \sum_{k:(j,k)\in E} P_{jk},\ j = 1, \ldots, n \quad (1a)$$

$$-q_j = Q_{ij} - x_{ij}\ell_{ij} - \sum_{k:(j,k)\in E} Q_{jk},\ j = 1, \ldots, n \quad (1b)$$

$$v_j = v_i - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}) + (r_{ij}^2 + x_{ij}^2)\ell_{ij}, \quad (i, j) \in E \quad (1c)$$

$$\ell_{ij} = \frac{P_{ij}^2 + Q_{ij}^2}{v_i}, \quad (i, j) \in E, \quad (1d)$$

where $\ell := |I_{ij}|^2$, $v_i := |V_i|^2$. Equations (1) define a system of equations in the variables $(P, Q, \ell, v) := (P_{ij}, Q_{ij}, \ell_{ij}, (i, j) \in E, i = 1, \ldots, n)$, which do not include phase angles of voltages and currents. Given an (P, Q, ℓ, v) these phase angles can be uniquely determined for radial networks. This is not the case for mesh networks; see [12] for exact conditions under which phase angles can be recovered for mesh networks.

B. Linear approximation of the branch flow model

Real distribution circuits usually have very small r, x, i.e. $r, x << 1$, while $v \sim 1$. Thus real and reactive power losses are typically much smaller than power flows P_{ij}, Q_{ij}. Following [7], we neglect the higher order real and reactive power loss terms in (1) by setting $\ell_{ij} = 0$ and approximate P, Q, v using the following linear approximation, known as Simplified Distflow introduced in [7].

$$-p_j = P_{ij} - \sum_{k:(j,k)\in E} P_{jk},\ j = 1, \ldots, n \quad (2a)$$

$$-q_j = Q_{ij} - \sum_{k:(j,k)\in E} Q_{jk},\ j = 1, \ldots, n \quad (2b)$$

$$v_j = v_i - 2(r_{ij}P_{ij} + x_{ij}Q_{ij}), (i, j) \in E \quad (2c)$$

From (2), we can derive that the voltage $v = (v_1, \ldots, v_n)^T$ and power injection $p = (p_1, \ldots, p_n), q = (q_1, \ldots, q_n)$ satisfy the following equation:

$$v = Rp + Xq + v_0 \quad (3)$$

where $R = [R_{ij}]_{n\times n}$, $X = [X_{ij}]_{n\times n}$ are given as follows:

$$R_{ij} := 2\sum_{(h,k)\in\mathcal{P}_i\cap\mathcal{P}_j} r_{hk},$$

$$X_{ij} := 2\sum_{(h,k)\in\mathcal{P}_i\cap\mathcal{P}_j} x_{hk}.$$

Here $\mathcal{P}_i \subset E$ is the set of lines on the unique path from bus 0 to bus i. The detailed derivation is given in [14]. Since $r_{ij} > 0, x_{ij} > 0$ for all i, j, R, X have the following properties.

Lemma 1. *R, X are positive definite and positive matrices.*[2]

Proof. We refer readers to [14] for the detailed proof. □

C. Problem formulation

Before rigorously formulating the Volt/Var control problem, we separate q into two part, $q^c = (q_1^c, \ldots, q_n^c)$ and $q^e = (q_1^e, \ldots, q_n^e)$, where q^c denotes the reactive power injection governed by the Volt/Var control components and q^e denotes any other reactive power injection.[3] Let $v^{par} \triangleq Rp + Xq^e + v_0$, then,

$$v = Xq^c + v^{par}.$$

The goal of Volt/Var control on a distribution network is to provision reactive power injections q^c to maintain the bus voltages v within a tight range $[\underline{v}, \bar{v}]$ under any operating condition given by v^{par}. Without causing any confusion, in

[2] A matrix is a positive matrix iff each item is positive.

[3] For easy exposition, we assume that there is a q_i^c at each bus i. But the algorithm extends to the scenario that only a subset of buses have Volt/Var control component.

the rest of the paper, we will simply use q_i instead of q_i^c to denote the reactive power pulled by the Volt/Var control. The Volt/Var control can be modeled as a control problem on a quasi-dynamical system with state v and controller q; that is, given the current state $v(t)$ and other available information, the controller determines a new reactive power injections $q(t)$ and the new $q(t)$ results in a new voltage profile $v(t+1)$ according to (3). Mathematically, the Volt/Var control problem is formulated as the following closed loop dynamical system,

$$v(t+1) = Xq(t) + v^{par}; \quad (4a)$$
$$q(t) = u(\text{information at time } t). \quad (4b)$$

where $u = (u_1, \ldots, u_n)$ is the Volt/Var controller. The objective of Volt/Var control is to design u to lead the system voltage $v(t)$ to reach the acceptable range $[\underline{v}, \bar{v}]$ under any system operating condition which is given by v^{par}. Mathematically, it requires that

$$\lim_{t\to\infty} \text{dist}(v(t), [\underline{v}, \bar{v}]) = 0.$$

Here $\text{dist}(y, z) := \min_{z\in Z} ||y - z||$ where y is a point and Z is a set. Note that (4a) is governed by the system intrinsic dynamics (Kirchoff's Law) and not able to controlled or tuned, which makes the Volt/Var control challenging.

The problem we will address in this paper is the information requirement of the controller u in order to stabilize the voltage in the acceptable range. In general, in electricity distribution networks, only a small portion of buses are monitored, individuals are unlikely to announce their generation or load profile, the availability of DERs are fluctuating and uncertain, and even the grid parameters and the topology are only partially known. All of these facts demand decentralized algorithms for the voltage control, i.e., each control component adjusts its reactive power input based on the local signals that are easy to measure or to communicate.

III. Voltage control using only voltage measurements: Impossibility result

We first study such Volt/Var control rules that use merely voltage measurements as the control information. This type of control has been proposed and discussed in many existing literature and applications. For example, the IEEE standard association proposes decentralized voltage control for inverters using the deviation of the local voltages from the nominal value [15], [16]: an inverter monitors its terminal voltage and sets its reactive power generation based on a static and predefined Volt/Var curve. The scheme is shown in Figure 1. Besides the proposed IEEE standards, there are other researches promoting adapting local power injection according to the voltage, e.g., [6], [8]. However, in the following, we show that this type of controller is insufficient for Volt/Var control.

In fact, we will demonstrate that as long as u is in the following form,

$$p(t) = u(v(t)), \quad (5)$$

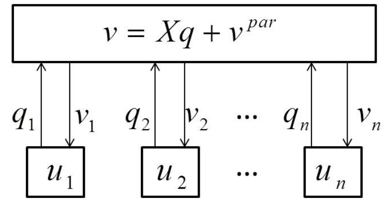

Fig. 1: One proposed IEEE standard: Decentralized Volt/Var control using local voltage measurements.

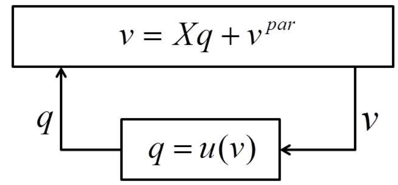

Fig. 2: Volt/Var control using merely voltage information: This type of controller is insufficient for Volt/Var control.

and u maps the bounded set $[\underline{v}, \bar{v}]$ to a bounded set, then it is impossible for such u to maintain acceptable voltages under all the operating condition, no matter whether u is in a centralized or decentralized form. This is formally stated in the following proposition.

Proposition 2. *For any u in the form of (5) that maps $[\underline{v}, \bar{v}]$ to a bounded set, there exist v^{par} such that this controller is not able to stabilize the voltage v in the acceptable range $[\underline{v}, \bar{v}]$.*[4]

Proof. The proof is straghtforward. Subsistuting (5) into (4a), we have:

$$v(t+1) - Xu(v(t)) = v^{par}.$$

Given a v^{par}, if u stabilizes the voltage in the acceptable range $[\underline{v}, \bar{v}]$, i.e., $\lim_{t\to\infty} \text{dist}(v(t), [\underline{v}, \bar{v}]) = 0$, then v^{par} should be at least in the set of $M := \{v - Xu(\tilde{v}) : v, \tilde{v} \in [\underline{v}, \bar{v}]\}$ which is bounded because u maps $[\underline{v}, \bar{v}]$ to a bounded set. Thus we know there exist v^{par} that the controller is not able to stabilize the voltage in the acceptable range. □

This proposition tells us that any Volt/Var control depending merely on voltage information is not suitable to maintain acceptable voltages, no matter whether it is decentralized or centralized. Thus we should consider adding (or using) other information to design controller u. In the rest of the paper, we will show that if we use both the information of the current q and v, then a fully decentralized in the form of $u_i(p_i(t), v_i(t))$ is able to maintain acceptable voltages; further, if we introduce some auxiliary variables, then a fully decentralized algorithm in the form of $u_i(\text{virtual variables at } i, v_i)$ is able to both maintain acceptable voltages and minimize a cost of reactive power compensation in a particular form.

[4] Note that any continuous function maps $[\underline{v}, \bar{v}]$ to a bounded set.

IV. A DECENTRALIZED ALGORITHM TO REACH THE ACCEPTABLE VOLAGE RANGE

In this section, we will show that by using both the local voltage and reactive power information, a fully decentralized algorithm in the form of $q_i(t) = u(v_i(t), q_i(t-1))$ is able to stabilize the voltage in the acceptable range. The scheme of the algorithm is shown in Figure 3.

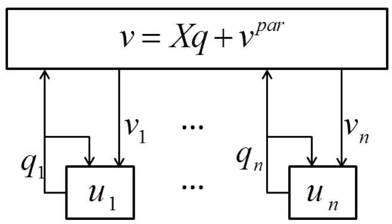

Fig. 3: Volt/Var control using local information of local voltage and reactive power injection: This type of controller is able to maintain acceptable voltages.

A. Algorithm

The decentralized algorithm is given as follows:

$$q_i(t) = q_i(t-1) - \epsilon d_i(v_i(t)) \tag{6}$$

where

$$d_i(v_i) := [v_i - \bar{v}_i]^+ - [\underline{v}_i - v_i]^+ \tag{7}$$

and ϵ are positive constant stepsizes. Here $[]^+$ is defined as $[a]^+ := \max(a, 0)$.

Algorithm in (6) says that if the local voltage at bus i is above its upper limit, bus i decreases the reactive power injection; in contrast, if the local voltage is below its lower limit, bus i increases its reactive power injection. This algorithm is very simple and intuitive, yet we will prove that regardless of v^{par}, the voltage will asymptotically converge to the acceptable range $[\underline{v}, \bar{v}]$ under the controller specified in (6). Before that, we discuss the properties of the algorithm (6) which make it attractive to real-time and scalable implementation.

i) We note that in the algorithm, each bus i uses only the local voltage measurement $v_i(t)$ and its previous reactive power injection $q_i(t-1)$, and the control is similar to an integral controller. Thus, the algorithm does not require any communication and the implementation is simple.
ii) The algorithm does not require any system operating information about v^{par}. This makes the algorithm practical because due to the large volatility and uncertainty of renewable energy, time-varying nature of uncertainty, and the privacy concern of consumers, v^{par} is not available and has huge uncertainty.
iii) Though the convergence of the algorithm depends on the step size value ϵ as shown in the next section, how to choose ϵ to guarantee the convergence is independent of v^{par}. As a result, once we have incorporated the algorithm into the hardware design of Volt/Var control, the Volt/Var control will work under any system operating operation.

B. Proof of the convergence

In this section, we prove that the voltage asymptotically converges to the acceptable range $[\underline{v}, \bar{v}]$ under the controller specified in (6). Before that, we introduce the function,

$$\phi(v) = \sum_{i=1}^{n} \left([v_i - \bar{v}_i]^+\right)^2 + \sum_{i=1}^{n} \left([\underline{v}_i - v_i]^+\right)^2.$$

Substituting $v = Xq + v^{par}$ into this function, ϕ also defines a function of q. Without causing any confusion, we will abuse the notation of ϕ, meaning that we define

$$\phi(q) := \phi(v(q)) = \phi(Xq + v^{par}).$$

The main idea of the proof is to show that $\phi(q(t))$ keeps decreasing along the trajectory of the algorithm (6) until the corresponding voltage $v(t)$ reaches the acceptable range $[\underline{v}, \bar{v}]$.

From simple derivation, we know that

$$\nabla \phi(q) = X \left[[v - \bar{v}]^+ - [\underline{v} - v]^+ \right] \tag{8}$$

where

$$\nabla \phi(q) := \left[\frac{\partial \phi}{\partial q_1}, \ldots, \frac{\partial \phi}{\partial q_n} \right]^T,$$

$$[v - \bar{v}]^+ := \left[[v_1 - \bar{v}_1], \ldots, [v_n - \bar{v}_n] \right]^T,$$

$$[\underline{v} - v]^+ := \left[[\underline{v}_1 - v_1]^+, \ldots, [\underline{v}_n - v_n]^+ \right]^T.$$

Notice that

$$\nabla \phi(q) = X \left[d_1(v_1), \ldots, d_n(v_n) \right]^T. \tag{9}$$

Lemma 3. *Let $L \triangleq \sigma^2_{\max}(X)$, then we have $\forall q, \tilde{q}$,*

$$\|\nabla \phi(q) - \nabla \phi(\tilde{q})\| \le L \|q - \tilde{q}\|.$$

Proof. From (8), we know that

$$\begin{aligned} \|\nabla \phi(q) - \nabla \phi(\tilde{q})\| &\le \sigma_{\max}(X) \left\| [v - \bar{v}]^+ - [\underline{v} - v]^+ \right. \\ &\quad \left. - \left([\tilde{v} - \bar{v}]^+ - [\underline{v} - \tilde{v}]^+ \right) \right\| \\ &\le \sigma_{\max}(X) \cdot \|v - \tilde{v}\| \\ &\le \sigma^2_{\max}(X) \cdot \|q - \tilde{q}\| \end{aligned} \tag{10}$$

□

Therefore, we have the following lemma about the monotonicity of $\phi(q(t))$.

Lemma 4. *If $0 < \epsilon \le \frac{2\sigma_{min}(X)}{L}$, then $\phi(q(t+1)) \le \phi(q(t))$ and the equality holds if and only if $d_i(v(t)) = 0$ for all $i \in N$.*

Proof. By using the descent lemma (Prop. A. 24 of [17]), we have

$$\phi(q(t+1)) - \phi(q(t)) \le -\epsilon \nabla \phi' d + \frac{1}{2} \epsilon^2 L ||d||^2.$$

Substituting (9) into the inequality, we have:

$$\begin{aligned}\phi(q(t+1)) - \phi(q(t)) &\leq -\epsilon d(v)^T X d(v) + \frac{1}{2}\epsilon^2 L||d(v)||^2 \\ &\leq \left(-\epsilon\sigma_{\min}(X) + \frac{1}{2}\epsilon^2 L\right)||d(v)||^2\end{aligned}$$

Therefore we know that as long as $0 < \epsilon \leq \frac{2\sigma_{min}(X)}{L}$,

$$\phi(q(t+1)) - \phi(q(t)) \leq 0,$$

and the equality holds if and only if $d(v) = 0$. □

Through simple derivation, we know that $d(v) = 0$ if and only if $v \in [\underline{v}, \bar{v}]$, as shown in the following.

Lemma 5. *$d_i(v_i) = 0$ if and only if $v_i \in [\underline{v}_i, \bar{v}_i]$.*

Proof. Note that $d_i(v_i) = [v_i - \bar{v}_i]^+ - [\underline{v}_i - v_i]^+$. Therefore, $d_i(v_i) = 0$ iff $[v_i - \bar{v}_i]^+ = [\underline{v}_i - v_i]^+$. This equality holds iff $[v_i - \bar{v}_i]^+ = [\underline{v}_i - v_i]^+ = 0$. The statement follows. □

Lemma 4 and Lemma 5 implies the following convergence of the algorithm (6).

Theorem 6. *If $0 < \epsilon \leq \frac{2\sigma_{min}(X)}{L}$, then $v(t)$ converges to the acceptable set $[\underline{v}, \bar{v}]$.*

Proof. Note that $\phi \geq 0$ for any q, and $\phi \to \infty$, if $q \to \infty$. From Lasalle Theorem [18], Lemma 4 and Lemma 5, we know that $v(t)$ converges to the acceptable set $[\underline{v}, \bar{v}]$. □

Before closing this section, we need to point out that the proof only demonstrates the convergence to the feasible set $[\underline{v}, \bar{v}]$ rather than a fixed point in this set.

V. A DECENTRALIZED ALGORITHM TO REACH AN OPTIMAL FEASIBLE POINT

The previous algorithm only guarantees the convergence to the feasible set $[\underline{v}, \bar{v}]$. If we introduce additional local auxiliary variables and design local updating rules using the auxiliary variables, we will be able to develop a decentralized algorithm which converges to a feasible point in $[\underline{v}, \bar{v}]$. This point also minimizes a cost of reactive power provision in a certain form. We first provide the algorithm and then discuss its convergence and the optimality of the equilibrium point.

For each bus i, we introduce two local auxiliary variables, $\bar{\lambda}_i$ and $\underline{\lambda}_i$. At each time t, given the local voltage measurement $v_i(t)$, the updating rule for $\bar{\lambda}_i, \underline{\lambda}_i, q_i(t)$ is given by:

$$\bar{\lambda}_i(t+1) = \left[\bar{\lambda}_i(t) + \epsilon(v_i(t) - \bar{v}_i)\right]^+ \quad (11a)$$
$$\underline{\lambda}_i(t+1) = \left[\underline{\lambda}_i(t) + \epsilon\left(\underline{v}_i - v_i(t)\right)\right]^+ \quad (11b)$$
$$q_i(t+1) = \underline{\lambda}_i(t+1) - \bar{\lambda}_i(t+1) \quad (11c)$$

where ϵ is the step size. The diagram of the algorithm is shown in Figure 4.

In this algorithm, the control of the local reactive power depends only on the two local auxiliary variables and the updating rule of the two variables depends on their own values and the current local voltage measurement. Moreover, the updating rule is also similar to an integral controller with saturation. As a result, this algorithm is fully decentralized

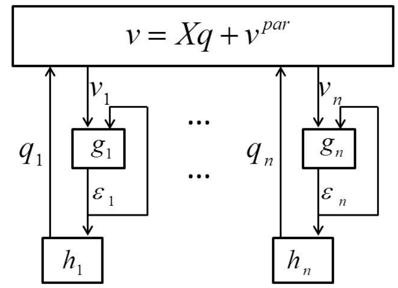

Fig. 4: Volt/Var control using local information of voltage, reactive power injection, and additional auxiliary variables: This type of controller is able to guarantee both of feasibility and optimality.

and it shares the same properties with the decentralized algorithm in (6), making it also attractive to practical implementation.

Further, as shown in the following theorem, not only does the algorithm converge to a feasible point, but it also converges to an optimal point which minimizes a cost of reactive power provision in a particular form.

Theorem 7. *If $\epsilon < \frac{1}{\sigma_{\max}}(X)$, $q(t)$ in the algorithm (11) converges to the optimal point q^* of the following optimization problem:*

$$\min_q \quad \frac{1}{2}q^T X q \quad (12a)$$
$$s.t. \quad Xq + v^{par} \leq \bar{v}, \quad (12b)$$
$$Xq + v^{par} \geq \underline{v}, \quad (12c)$$

and $v(t)$ converges to the corresponding voltage $v^* = Xq^* + v^{par}$, which is in the acceptable range $[\underline{v}, \bar{v}]$.

Proof. Introducing dual variables to the optimization problem (13), we have the following dual gradient method [17]:

$$\begin{aligned}\bar{\lambda}(t+1) &= \left[\bar{\lambda}(t) + \epsilon(Xq(t) + v^{par} - \bar{v})\right]^+ \\ \underline{\lambda}(t+1) &= \left[\underline{\lambda}(t) + \epsilon\left(\underline{v} - Xq(t) - v^{par}\right)\right]^+ \\ q(t+1) &= \arg\min_q \{\frac{1}{2}q^T X q + \bar{\lambda}(t+1)(Xq + v^{par} - \bar{v}) \\ &\quad + \underline{\lambda}(t+1)(\underline{v} - Xq - v^{par})\}\end{aligned}$$

The preceding algorithm is equivalent to the following:

$$\begin{aligned}\bar{\lambda}(t+1) &= \left[\bar{\lambda}(t) + \epsilon(v(t) - \bar{v})\right]^+ \\ \underline{\lambda}(t+1) &= \left[\underline{\lambda}(t) + \epsilon\left(\underline{v} - v(t)\right)\right]^+ \\ q(t+1) &= \underline{\lambda}(t+1) - \bar{\lambda}(t+1)\end{aligned}$$

which is exactly the algorithm (11).

Through simple derivation, the dual problem of (13) is given by:

$$\max_{\bar{\lambda}\geq 0, \underline{\lambda}\geq 0} -\frac{1}{2}(\underline{\lambda} - \bar{\lambda})X(\underline{\lambda} - \bar{\lambda}) + \bar{\lambda}(v^{par} - \bar{v}) + \underline{\lambda}(\underline{v} - v^{par})$$

Let $\tilde{X} = \begin{bmatrix} X & -X \\ -X & X \end{bmatrix}$. Then we have $\sigma_{\max}(\tilde{X}) = 2\sigma_{\max}(X)$. Therefore, we know that if $\epsilon < \frac{2}{\sigma_{\max}(\tilde{X})} =$

$\frac{1}{\sigma_{\max}(X)}$, $\bar{\lambda}(t), \underline{\lambda}(t)$ converge to the dual optimum. Correspondingly, $q(t)$ converges to the optimal point of (13). The conclusion of the theorem follows. □

VI. Case Study

In this section we evaluate the two decentralized algorithms on a distribution circuit of South California Edison with a high penetration of photovoltaic (PV) generation [19]. Figure 5 shows a 56-bus distribution circuit. Note that Bus 1 indicates the substation, and there are 1 photovoltaic (PV) generators located on buses 45 and there are shunt capacitors located at bus 19, 21, 30, 53. See [19] for the network data including line impedance, peak MVA demand of loads and the nameplate capacity of the shunt capacitors and the photovoltaic generation.

In the simulation, we assume that there are Volt/Var control components at bus 19, 21, 30, 45, and 53 and those control components can pull in (supply) and out (consume) reactive power. The nominate voltage magnitude is 12kV and the acceptable range is set as $[11.4\text{kV}, 12.6\text{kV}]$ which is the plus/minus 5% of the nominate value. Though the analysis of this paper is built on the linearized power flow model (2), we simulate the voltage control algorithms (6,11) using the full nonlinear AC power flow model (1).

We simulates three different scenarios: 1) the distribution feeder is supplying heavy loads without any PV generation, resulting in low voltages at some buses (Figure 6); 2) the PV generator is generating a large amount of power but the loads are moderate, resulting in high voltages at some buses (Figure 7); 3) the PV generator is generating a large amount of power and some buses are having heavy loads, resulting in high voltages at some buses and low voltages at other buses (Figure 8). All of the simulation results demonstrates that both of the two algorithms converge very fast. The algorithm in (11) takes a couple of more steps to stabilize but it stabilizes at the point with less absolute reactive power provision.

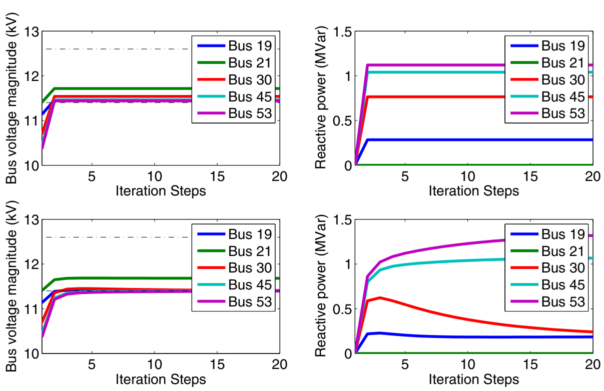

Fig. 6: Heavy loads, no PV generation: The upper figures shows simulation results using the decentralized algorithm in Section IV and the lower figures shows simulation results using the decentralized algorithm in Section V.

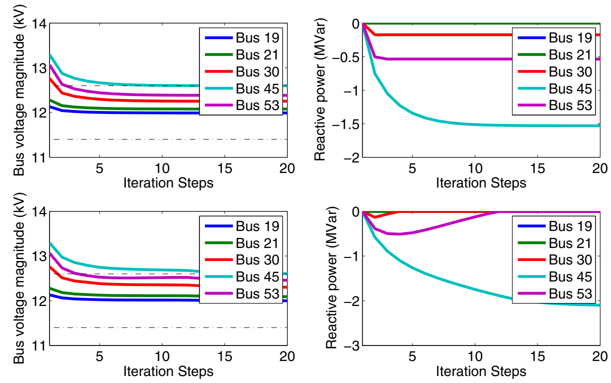

Fig. 7: Moderate loads, large PV generation: The upper figures shows simulation results using the decentralized algorithm in Section IV and the lower figures shows simulation results using the decentralized algorithm in Section V.

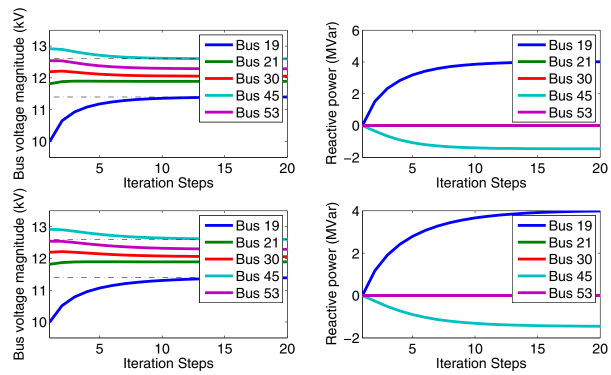

Fig. 8: Heavy loads, large PV generation: The upper figures shows simulation results using the decentralized algorithm in Section IV and the lower figures shows simulation results using the decentralized algorithm in Section V.

VII. Conclusion

In this paper, we study how different information structures affect the performance of Volt/Var control. In particular, we first show that using only voltage measurements to decide reactive power injection is insufficient to maintain acceptable voltages. Then we proposes two fully decentralized algorithms by adding additional information into the control design. Both of them can maintain acceptable voltages; but one is also able to optimize the reactive power injection in terms of minimizing a cost of the reactive power compensation. Both of the two algorithms uses only local measurements and local variables, requiring no communication.

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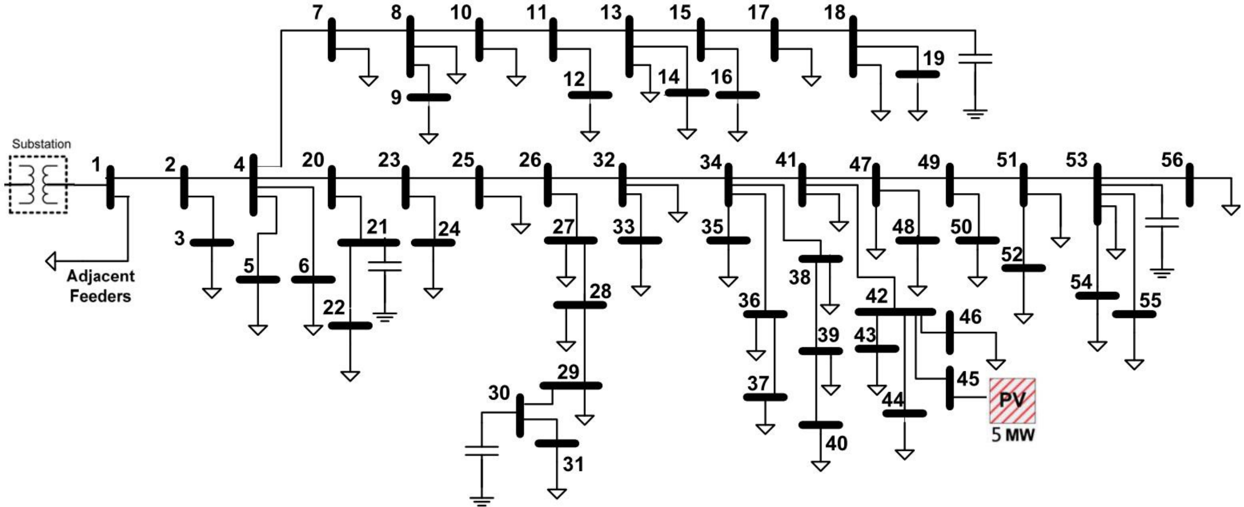

Fig. 5: Schematic diagram of two SCE distribution systems.

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