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An Incentive-Based Approach to Distributed Estimation with Strategic Sensors

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Abstract— **Consider a setup in which a central estimator seeks to estimate a random variable using measurements from multiple sensors. The sensors incur an effort cost through consumption of resources to obtain a measurement. The sensors are self-interested and need to be compensated to generate measurements with a low enough error covariance that allows the calculation of an estimate with sufficient accuracy. However, a simple compensation scheme based on self-reported effort taken will not be sufficient, since both the quality of measurements taken by a sensor and the measurement values are private information for the sensor. A strategic sensor can misreport these values to increase his compensation. We formulate this problem as a contract design problem between the sensors and the central estimator and present an optimal contract between the central estimator and sensors as the solution.**

I. INTRODUCTION

Smart personal devices equipped with a rich array of sensors have enabled the dawn of participatory or crowd sensing. In this paradigm, a large number of sensors owned by different individuals generate measurements about the environment and share these data with a central estimator. The central estimator collects the data and generates the required estimate of the environment being monitored. Given that the sensors do not belong to the central estimator, the quality of the estimate generated through participatory sensing depends on the willingness of the individual owners to generate and submit accurate information. On the other hand, to sense and transmit data, sensors must invest effort (e.g. battery power). While many existing systems rely on individuals doing so voluntarily, it is likely that such voluntary participation will decline without sufficient monetary incentives to motivate individuals to participate in data collection, once the novelty has worn off.

Given its importance, the problem of providing incentives for participation in crowd sensing has been considered in many papers. A review of various incentive based mechanisms, including both monetary and non-monetary incentives, is provided in [1] and [2]. A variety of incentives have been discussed more specifically, including using micro-payment [3], reputation [4], [5], and auctions [6], [7]. Of particular relevance to the setting of this paper is [8], in which the authors consider a scenario where a central planner employs several sensors, who are selfish but truthful. Under the assumption of the total budget for compensating the sensors being limited, the authors provide a compensation scheme and analyze the impact of the total budget availability on estimate accuracy.

However, most of the existing literature assumes that sensors are not strategic, meaning that they do not anticipate the effects of their reporting on the compensation they would obtain. Hence, they do not provide false information about the effort they expended or the data they collected to gain more compensation. In this paper, we consider strategic sensors who can transmit false information to the central estimator.

In the setting, we model the problem as one of contract design, and provide a contract by which strategic sensors (more precisely, their owners) can be suitably incentivized to participate, and further both generate and report high quality measurements even if doing so will require a higher consumption of resources such as battery power. For simplicity, we will use the terms sensors and owners interchangeably.

A related problem is that of cheap talk in game theory (see [9] for a review and [10] for a specific application in a setting related to ours). In this problem, a strategic sensor who can transmit false information is considered; however, it is assumed that both the central estimator and the strategic sensor share the objective of minimizing the estimation error. We consider a scenario in which the interests of the sensor and central estimator are misaligned. The sensor is interested not in minimizing the error at the estimator but in maximizing his compensation, given that he incurs effort cost for taking measurement of a specified quality. On the other hand, the objective of the central estimator is to obtain an estimate with a guaranteed quality while minimizing the payment to the sensor.

We formulate this problem as a game between the sensors and the central estimator, and present an optimal contract between the central estimator and the sensors. The contract has to satisfy various constraints:

- First, the contract should incentivize sensors to participate, i.e, each sensor should have an expected utility through participation that is no less than that gained by not participating.
- Second, given that the central estimator does not know the actual effort that the sensors make, and hence the measurement quality, a strategic sensor can misreport the quality to seek a higher payment. The contract, therefore, should minimize the payment to sensors while guaranteeing the estimate accuracy.

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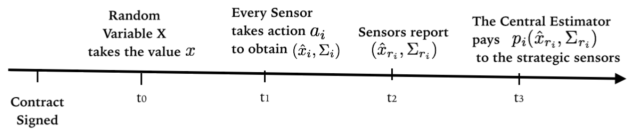

Fig. 1. Timeline of the proposed problem.

- Third, the estimate at the central estimator must have guaranteed quality in terms of the error covariance. Thus, the sensors need to be incentivized to generate measurements of high enough quality even if it requires larger consumption of resources.

The main contribution of our work is to formulate the problem as a contract design problem and present a payment scheme that compensates the strategic sensor based on two criteria: first, the quality of the estimate the strategic sensor transmits, and second, how close that estimate is to the estimate of other sensors. One interesting result is that under some assumptions, the proposed payment leads to strategic sensors communicating truthfully as an equilibrium. Further, the optimal level of effort each sensor invests and the quality of the estimate it transmits improves as the quality of estimate provided by the other sensors decreases.

The rest of the paper is organized as follows. In Section II, the problem statement is presented. In Section III, we begin with a contract structure for the case when there is only one strategic sensor present and study the optimal effort invested by the sensor in response. Next, we generalize the contract to the case when multiple strategic sensors are present. In Section IV, we conclude the paper and present some avenues for future work.

Notation: : $f_{X|Y}(x|y)$ (which is sometimes simplified to $f(x|y)$ for notational simplicity) denotes the probability distribution function (pdf) of random variable X given that another random variable Y assumes value y. The Gaussian pdf is denoted by $\mathcal{N}(m, \sigma^2)$ where m is the mean and σ is the standard deviation. The expectation of a random variable X is shown by $\mathbb{E}[X]$. By abusing notation, we sometimes write the expectation as $\mathbb{E}[x]$. Let $R_{XY} = \mathbb{E}[XY]$ denote the correlation between two random variables X and Y, sometimes written as R_{xy}. All variables are real-valued unless mentioned otherwise.

II. Problem Statement

We consider Bayesian static estimation in a centralized setting with multiple sensors. Consider a random variable $X \sim \mathcal{N}\left(0, \sigma_x^2\right)$ that assumes the value x in an experiment. A central estimator wishes to obtain an estimate $\hat{x}_g$ (the subscript g stands for 'global') of x in the minimum mean squared error (MMSE) sense. To this end, it obtains estimates based on local measurements taken by each of N sensors. Each measurement from any sensor consumes resources from the sensor. Each sensor i generates a local MMSE estimate $\hat{x}_i$ of x with error covariance Σ_i using its own measurements. Denote the estimation error in estimates $\hat{x}_i$ by e_i, so that

$$\hat{x}_i = x - e_i \quad e_i \sim \mathcal{N}(0, \Sigma_i). \tag{1}$$

The error covariance can be reduced if the sensor decides to expend more resources; e.g., by taking more number of measurements or by taking more accurate measurements. We denote the choice of resource consumption by the sensor i as the sensor choosing an effort a_i.

Assumption 1: The local error covariance Σ_i is a decreasing, convex and twice continuously differentiable function $f_i(a_i)$ of the effort a_i. Further, the functional form f_i is public knowledge, while a_i is private knowledge for sensor i.

To make clear the dependence on a_i, we sometimes use the notation $\Sigma_i(a_i)$. Further, we assume that the local estimate $\hat{x}_i$ is generated using a linear measurement of the form

$$y_i = C_i x + v_i, \quad v_i \sim \mathcal{N}(0, \sigma_i^2) \tag{2}$$

where the effect of the effort a_i may be captured, e.g., by the covariance of the measurement noise v_i or the dimension of the vector y_i.

We assume that errors in all the local estimates are independent of each other and independent of X. Thus, if the central estimator had access to the local estimates and error covariances, it could fuse them to calculate the global estimate. Further, if the estimator wanted to generate an estimate with the error covariance below a specified bound, it could ask the sensors to expend sufficient effort and then compensate them proportional to the effort they expend. However, such a compensation scheme may incentivize the sensors to misreport the effort they expended, and hence lie about their estimates and error covariances to gain higher compensation if possible.

Denote by $\hat{x}_{r_i}$ and Σ_{r_i} the estimate and the error covariance values the sensor i reports to the central estimator at t_2. Based on the received estimate and error covariance values, the central estimator calculates the global estimation $\hat{x}_g$ with error covariance Σ_g, and pays payment p_i to each sensor i. Thus, the timeline of the problem is as shown in Figure 1. We make the following assumptions

1) System parameters such as σ_x and N are known to all the sensors and the central estimator. However, the estimate value and the corresponding error covariance of each sensor are private information for that particular sensor. Sensors do not know about the decision of other sensors. Hence, sensors take independent decision.
2) All sensors are rational and they send (incorrect) information such that their profit is maximized.
3) One out of the N sensors is *'loyal'* in the sense that it reports $\hat{x}_{r_i} = \hat{x}_i$ and $\Sigma_{r_i} = \Sigma_i$. Notice that this sensor can correspond to a priori information about x available to the central estimator and may not be a physical sensor.
4) The identity of the loyal sensor and what she transmits are known to the central estimator but not to the other sensors.

We now formulate the utility functions and the strategy spaces of the various decision makers.

- Each sensor i has a utility function that consists of three terms:

$$U_i = -h(a_i) - g\left(\hat{x}_{r_i} - \hat{x}_i, \Sigma_{r_i} - \Sigma_i\right) + p_i, \qquad (3)$$

where $h(a_i)$ models the effort cost which is an increasing function of the effort a_i taken by the sensor, $g\left(\hat{x}_{r_i} - \hat{x}_i, \Sigma_{r_i} - \Sigma_i\right)$ is the falsification cost that models the cost for the sensor to report $\hat{x}_{r_i}$ and Σ_{r_i} instead of the true local estimate $\hat{x}_i$ and error covariance Σ_i, and p_i is the payment from the central estimator. Note that both $\hat{x}_i$ and Σ_i are functions of a_i. The falsification cost is an non-decreasing function in the absolute value of either argument $\hat{x}_{r_i} - \hat{x}_i$ or $\Sigma_{r_i} - \Sigma_i$; note that $g(.)$ is considered for generality and can be zero.

Assumption 2: For concreteness, we model the effort cost and the falsification cost by quadratic functions; thus, $h(a_i) = \frac{{a_i}^2}{2}$ and $g\left(\hat{x}_{r_i} - \hat{x}_i, \Sigma_{r_i} - \Sigma_i\right) = \frac{(\hat{x}_{r_i} - \hat{x}_i)^2}{2} + \frac{(\Sigma_{r_i} - \Sigma_i)^2}{2}$.

Each sensor i wishes to maximize her expected utility $\mathbb{E}[U_i]$ over choice of the triplet $\{a_i, \hat{x}_{r_i}, \Sigma_{r_i}\}$. Notice that, by definition, for the loyal sensor $\hat{x}_{r_i} = \hat{x}_i$ and $\Sigma_{r_i} = \Sigma_i$.

- The objective of central estimator is to obtain an estimate of high quality, as measured by low error covariance Σ_g, while making the minimum payment possible. For simplicity, we assume the optimization problem for the central estimator to be

$$\min\{\mathbb{E}[P]\}$$
$$\text{while ensuring that } \Sigma_g \text{ is below a specified bound,}$$

where Σ_g is the global error covariance of the global estimate $\hat{x}_g$ and $P = \sum_{i=1}^{N} p_i$ is the total payment made to the sensors. The action space of the central estimator is how to calculate the global estimate $\hat{x}_g$ and the payments p_i based on received information $\hat{x}_{r_i}$ and Σ_{r_i} as a function of the contract for various sensors.

We assume that the central estimator solves the optimization problem in two steps. In the first step, we focus on minimizing the payments through the design of an optimal contract and calculate the error covariance Σ_g that results. This yields the minimum (expected) payment as a function of Σ_g. In the second step, we can then calculate for a given bound on Σ_g what is the minimum payment that is needed. We concentrate on the first step of this procedure since the second step requires specific assumptions on the functional form of the dependence of the accuracy of the local estimates on the effort by the sensors.

In the first step, the optimization problem for the designer can be stated as

$$\min_{p_i, \hat{x}_g} \mathbb{E}\left[P\right] \qquad (4)$$

subject to

(i) rational actions by each sensor i to maximize $\mathbb{E}\left[U_i\right]$
(ii) individual rationality constraints being satisfied for the strategic sensors; $\mathbb{E}[U_i] > 0$
(iii) Σ_g being calculated optimally based on the received data

where the expectation in (4) is taken with respect to the randomness inherent due to random variable X and the errors in the local estimates.

Remark 1: If the identity of the loyal sensor is known, the payment p_i to the loyal sensor can be considered to be fixed. Thus, we can redefine the payment made by the central estimator as being summed over all the non-loyal sensors. Further, we can pose the problem (4) in terms of this payment and the constraints (i) and (ii) as being valid only for the non-loyal sensors.

III. Contract Design

There is a fundamental misalignment between the utilities of the sensors and the central estimator. The payments p_i seek to align these utilities. The contract specifies these payments. First, we note some payment schemes that will not work.

- A payment scheme $p_i = c$ for a constant c that does not depend on the reported quantities will lead to each sensor i not making any effort and reporting some arbitrary value to the central estimator. In economics terms, this is termed as the problem of *moral hazard* [11, Chapter 4].
- A contract that specifies $p_i = f\left(\Sigma_{r_i}\right)$ where $f(.)$ is a decreasing function can be considered to incentivize the sensors to choose a higher a_i. However, this contract will incentivize a strategic sensor to report a very low error covariance Σ_{r_i} (through lying) irrespective of the actual value of a_i and Σ_i. This is the problem of *adverse selection* [11, Chapter 3].

A rough description of our approach is to design a payment scheme that has two parts (inspired, e.g., by [12], [13]). The first part depends on the self-reported value of the error covariance Σ_{r_i}, while the second depends on information independently obtained from the loyal sensor that incentivizes the strategic sensor not to misreport too much.

Remark 2: The second part of the payment requires the presence of a possibly noisy loyal sensor. While a sensor that can fit this role is present in most crowd sensing applications, this role can also be filled by a priori information about x. Note that since $P > 0$, the error covariance Σ_g must be better that obtained by using the loyal sensor alone.

Remark 3: Notice that the central estimator cannot design a compensation that directly depends on a_i, $\hat{x}_i$ and Σ_i, since these values are private information for the strategic sensor and not directly observable by the central estimator.

A. A Particular Case

For pedagogical ease, we begin with the particular case when $N = 2$. Thus, there are two sensors, with sensor 1 being loyal (without loss of generality) and sensor 2 being strategic. For ease of notation, we will denote the quantities for the loyal sensor by subscript l, so that $\hat{x}_l \triangleq \hat{x}_1$, $y_l \triangleq y_1$, $C_l \triangleq C_1$, $\sigma_l \triangleq \sigma_1$ and $\Sigma_l \triangleq \Sigma_1$. Similarly, we use $a_s \triangleq a_2$, $\hat{x}_s \triangleq \hat{x}_2$, $\Sigma_s \triangleq \Sigma_2$, $y_s \triangleq y_2$, $C_s \triangleq C_2$, $\sigma_s \triangleq \sigma_2$, $\hat{x}_r \triangleq \hat{x}_{r_2}$ and $\Sigma_r \triangleq \Sigma_{r_2}$.

1) Contract Structure: The payment for the loyal sensor is assumed to be fixed and the payment for only the strategic sensor needs to be specified. As discussed earlier, we propose a contract with two parts: the first part $p^1(\Sigma_r)$ of the payment depends on the self-reported value of the error covariance Σ_r, and the second, $p^2(\hat{x}_r, \hat{x}_l)$ depends on the information obtained from both the sensors. Thus, we propose the following payment function:

$$p_s = p^1(\Sigma_r) + p^2(\hat{x}_r, \hat{x}_l), \tag{5}$$

with

$$p^1(\Sigma_r) = c_1 - \alpha_1\Sigma_r, \quad p^2(\hat{x}_r, \hat{x}_l) = c_2 - \alpha_2(\hat{x}_r - \hat{x}_l)^2,$$

where α_1, α_2, c_1 and c_2 are positive parameters to be designed. Define $c \triangleq c_1 + c_2$. An interpretation of the payment function is as follows. The term $p^1(\Sigma_r)$ incentivizes the sensor to make a higher effort and obtain a lower error covariance Σ_s. However, the sensor can misreport lower Σ_r to gain more compensation. The term $p^2(\hat{x}_r, \hat{x}_l)$ incentivizes the strategic sensor to report truthfully by 'tethering' the reported estimate with the noisy version of the ground truth.

Remark 4: Note that even with no misreporting, the estimate $\hat{x}_s$ would not, in general, be equal to $\hat{x}_l$.

With this contract, the utility of the strategic sensor can be written as

$$U_s = -h(a_s) - g\left(\Sigma_r - \Sigma_s, \hat{x}_r - \hat{x}_s\right) + c - (\alpha_1\Sigma_r + \alpha_2(\hat{x}_l - \hat{x}_r)^2). \tag{6}$$

We make the following assumption:

Assumption 3: Σ_r is only a function of Σ_s and $\hat{x}_s$. In other words, misreporting the error covariance is decided based only on the true quantities.

2) Designing and characterizing the optimal contract: In this section, we solve the problem (4) with the utility of the strategic sensor given by (6). Our solution is in three steps. First we calculate the optimal value of $\hat{x}_r$ and Σ_r that are transmitted by the sensor. Then, we calculate the optimal effort taken by the sensor. Finally, we optimize the value of α_1, α_2 and c.

a) Optimal choice of $\hat{x}_r$, Σ_r and $\hat{x}_g$: We first design the optimal reporting function for the strategic sensor and estimate calculated at the central estimator. We begin with the following preliminary results that are proved in the Appendix.

Lemma 1: Consider the problem formulation considered in Sections II and III.

(i) The random variables $\hat{x}_l$ and $\hat{x}_s$ are zero-mean Gaussian random variables.

(ii) $\mathbb{E}[(x - \hat{x}_l)(x - \hat{x}_s)] = k\sigma_x^2$, $k = [\frac{\sigma_l^2}{C_l^2\sigma_x^2+\sigma_l^2}][\frac{\sigma_s^2}{C_s^2\sigma_x^2+\sigma_s^2}]$.

(iii) $E[\hat{x}_l|\hat{x}_s] = (\sigma_x^2 + k\sigma_x^2 - \Sigma_l - \Sigma_s)(\sigma_x^2 - \Sigma_s)^{-1}\hat{x}_s$.

Proof: See [15]. ■

Theorem 1: For the optimal contract that leads to the utility of the strategic sensor as in (6), the following holds:

(i) $\hat{x}_r = a\hat{x}_s$, $a = \frac{(1+2\alpha_2(\sigma_x^2+k\sigma_x^2-\Sigma_l-\Sigma_s)(\sigma_x^2-\Sigma_s)^{-1})}{1+2\alpha_2}$.

(ii) $\Sigma_r = \Sigma_s - \alpha_1$

(iii) $\Sigma_g^{-1}\hat{x}_g = {\Sigma_l}^{-1}\hat{x}_l + {\Sigma_s}^{-1}\hat{x}_s, \Sigma_g^{-1} = {\Sigma_l}^{-1} + {\Sigma_s}^{-1} - \sigma_x^{-2}$

Proof: To prove (i), consider the expected utility for the strategic sensor. We have

$$\begin{aligned}\mathbb{E}[U_s] &= \int\int [p_s - h(a_s) - g] f(\hat{x}_s, \hat{x}_l) d\hat{x}_l d\hat{x}_s \\ &= \int_{\hat{x}_s} f(\hat{x}_s) d\hat{x}_s \int_{\hat{x}_l} [p_s - h(a_s) - g] f(\hat{x}_l|\hat{x}_s) d\hat{x}_l.\end{aligned}$$

Note that $\hat{x}_r$ is a function of $\hat{x}_s$ and given Assumption 3, Σ_r is a function of Σ_s and $\hat{x}_s$ only. Thus, we can evaluate.

$$\begin{aligned}\frac{\partial\mathbb{E}[U_s]}{\partial\hat{x}_r} &= \int_{\hat{x}_s} f(\hat{x}_s)d\hat{x}_s \int_{\hat{x}_l} (\frac{\partial[p_s - h(a_s) - g]}{\partial\hat{x}_r}) f(\hat{x}_l|\hat{x}_s)d\hat{x}_l \\ &= \int_{\hat{x}_s} f(\hat{x}_s)d\hat{x}_s \int_{\hat{x}_l} [-(\hat{x}_r - \hat{x}_s) - 2\alpha_2(\hat{x}_r - \hat{x}_l)] f(\hat{x}_l|\hat{x}_s)d\hat{x}_l \\ &= -\int_{\hat{x}_s} f(\hat{x}_s)d\hat{x}_s \int_{\hat{x}_l} [(2\alpha_2 + 1)\hat{x}_r - \hat{x}_s - 2\alpha_2\hat{x}_l] f(\hat{x}_l|\hat{x}_s)d\hat{x}_l \\ &= -\int_{\hat{x}_s} f(\hat{x}_s)d\hat{x}_s [(2\alpha_2 + 1)\hat{x}_r - \hat{x}_s - \int_{\hat{x}_l} 2\alpha_2\hat{x}_l f(\hat{x}_l|\hat{x}_s)d\hat{x}_l] \\ &= -\int_{\hat{x}_s} f(\hat{x}_s)d\hat{x}_s \{(2\alpha_2 + 1)\hat{x}_r - \hat{x}_s - 2\alpha_2 E[\hat{x}_l|\hat{x}_s]\}.\end{aligned}$$

Setting this derivative equal to 0 to obtain the optimal choice of $\hat{x}_r$ yields

$$(2\alpha_2 + 1)\hat{x}_r(\hat{x}_s) - \hat{x}_s - 2\alpha_2\mathbb{E}[\hat{x}_l|\hat{x}_s] = 0$$

$$\Rightarrow \hat{x}_r = \frac{\hat{x}_s + 2\alpha_2\mathbb{E}[\hat{x}_l|\hat{x}_s]}{1 + 2\alpha_2}. \tag{7}$$

Given Lemma 1 (iii), we can write

$$\hat{x}_r = \frac{(1 + 2\alpha_2(\sigma_x^2 + k\sigma_x^2 - \Sigma_l - \Sigma_s)(\sigma_x^2 - \Sigma_s)^{-1})\hat{x}_s}{1 + 2\alpha_2}. \tag{8}$$

To prove (ii), we can similarly evaluate the derivative of the expected utility with respect to Σ_r and set it equal to zero, to obtain

$$\frac{\partial\mathbb{E}[U_s]}{\partial\Sigma_r} = -(\Sigma_r - \Sigma_s) - \alpha_1 = 0,$$

so that $\Sigma_r = \Sigma_s - \alpha_1$. Note that for $\Sigma_r > 0$, the strategic sensor must generate $\Sigma_s > \alpha_1$.

To prove (iii), we use the fact that the MMSE estimate at the central estimator can be obtained by combining the estimates from the loyal and strategic sensors. Thus, $\hat{x}_g$ is given by [14, p.103]

$$\Sigma^{-1}{}_g\hat{x}_g = {\Sigma_l}^{-1}\hat{x}_l + {\Sigma_s}^{-1}\hat{x}_s, \quad \Sigma^{-1}{}_g = {\Sigma_l}^{-1} + {\Sigma_s}^{-1} - \sigma_x^{-2}, \tag{9}$$

where $\Sigma_s = \Sigma_r + \alpha_1$ and $\hat{x}_s = a^{-1}\hat{x}_r$, given (ii). ■

Corollary 1: Consider the case when $\sigma_x^2 \to \infty$. Then, $\mathbb{E}[(x - \hat{x}_l)(x - \hat{x}_s)] = 0$ and the proposed payment (5) by the central estimator induces 'truth telling' by the strategic sensor in the estimate value $\hat{x}_r$.

Proof: See [15]. ■

b) Choice of the optimal effort: We now find the optimal effort that is induced for the strategic sensor. In the following, for simplicity, we assume the case when $\sigma_x^2 \to \infty$.

Theorem 2: The optimal effort a_s is given by $a_s{}^* = -(\alpha_2 + \alpha_1)\frac{d\Sigma_s}{da_s}$. Further, the optimal action is an increasing function of α_1 and α_2.

Proof: See [15]. ■

c) The optimal contract: We now optimize the value of α_1 and α_2 for the central estimator to solve problem (4). The CE seeks to choose α_1 and α_2 to minimize its payment to the strategic sensor (the payment to the loyal sensor is fixed). Thus, if we define $\bar{\alpha} = \{\alpha_1, \alpha_2\}$, then $\bar{\alpha}$ is given by

$$\bar{\alpha} = \arg\min \mathbb{E}[p_s] = \arg\min[c - (\alpha_1(\Sigma_s - \alpha_1) - \alpha_2(\Sigma_l + \Sigma_s)] \tag{10}$$

We continue to assume that $\sigma_x^2 \to \infty$, so that irrespective of the choice of α_1 and α_2, $\hat{x}_r = \hat{x}_s$. Since the optimal effort a_s^* can also be calculated by the central estimator, the true sensor error covariance Σ_s is known to the estimator. Thus, using Theorem 1 (iii), the central estimator can calculate the global error covariance as $\Sigma_g^{-1} = \Sigma_l{}^{-1} + \Sigma_s{}^{-1} - \sigma_x^{-2}$. Since Σ_g does not depend on Σ_r, we do not need the sensor to report Σ_r. In other words, we have the following result.

Theorem 3: In an optimal contract proposed in (5), $\alpha_1 = 0$.

Proof: Please see [15]. ■

Given $\alpha_1 = 0$, we can set $\Sigma_r = \Sigma_s$ and $\mathbb{E}[p_s] = c - \alpha_2(\Sigma_l + \Sigma_s)$.

Proposition 1: The optimal value of α_2 is given by the solution of equation $\alpha_2 \frac{d\Sigma_s}{d\alpha_2} = -(\Sigma_l + \Sigma_s)$. There exists a unique solution of the equation if $\alpha_2 \frac{d^2\Sigma_s}{d\alpha_2^2} + 2\frac{d\Sigma_s}{d\alpha_2} < 0$.

Proof: To optimize the value of α_2, we set $\frac{\partial \mathbb{E}[p_s]}{\partial \alpha_2} = 0$, which yields

$$\alpha_2 \frac{d\Sigma_s}{d\alpha_2} = -(\Sigma_l + \Sigma_s). \tag{11}$$

Now consider $\frac{\partial^2 \mathbb{E}[p_s]}{\partial {\alpha_2}^2} = -\alpha_2 \frac{d^2\Sigma_s}{d\alpha_2^2} - 2\frac{d\Sigma_s}{d\alpha_2} > 0$.

The right hand side is positive, if the condition in the statement of the Proposition holds. This implies that $\mathbb{E}[p_s]$ is a convex function of α_2, and hence, the minimizer exists and is unique. ■

d) Characterization of the optimal contract: We characterize some properties of the optimal solution below. The first result shows that if the loyal sensor transmits a more noisy estimate, the strategic sensor will expend higher effort, so that it can obtain an estimate of X with a lower error covariance. This result is interesting in that it characterizes how the optimal contract forces the strategic sensor to consider the 'total' noise faced by both the sensors.

Theorem 4: Assume that $\alpha_2 \frac{d^2\Sigma_s}{d\alpha_2^2} + 2\frac{d\Sigma_s}{d\alpha_2} < 0$, so that a unique optimal α_2 (and hence the payment p_s) exists. Then, Σ_s is a decreasing function of Σ_l.

Proof: See [15]. ■

The next result shows that the optimal contract derived in Theorem 2 and 3 satisfies the individual rationality constraints (ii) in the optimization problem (4). Recall that the constraint states that a sensor will participate in the estimation problem only if expected utility it hopes to obtain by participating in the contract is higher than expected utility without participation which is considered to be zero; $E[U_s] > 0$.

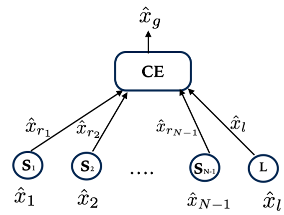

Fig. 2. Multiple sensor case.

Proposition 2: The proposed contract structure in (5) is ex ante individually rational, if $c > \alpha_2{}^*(\Sigma_l + \Sigma_s(a_s^*)) + \frac{a_s^{*2}}{2}$ where $\alpha_2{}^*$ and $a_s{}^*$ are designed as in Theorem 2 and 3. Denote by α_2^* the optimal choice of α_2.

Proof: If the contract is individually rational, we have

$$\begin{aligned}\mathbb{E}[U_s] &= \mathbb{E}[p_s] - h(a_s^*) > 0 \\ &= \frac{-a^{*2}}{2} + c - \alpha_2{}^*(\Sigma_l + \Sigma_s(a_s^*)) > 0\end{aligned}$$

which yields $c > \alpha_2{}^*(\Sigma_l + \Sigma_s(a_s^*)) + \frac{a_s^{*2}}{2}$. Since the parameter c is under the control of the designer, the individual rationality constraint can always be satisfied. ■

Remark 5: The discussion so far assumed that the loyal sensor is identified a priori. If the identity of the loyal sensor is unknown to the central estimator, a similar development can be done. In this case, the central estimator must pay both the sensors using the same contract, i.e, proportional to the estimation difference, $(\hat{x}_{r_1} - \hat{x}_{r_2})^2$. It is clear that the same results as above will hold since, by definition, the loyal sensor does not misreport irrespective of the compensation structure.

B. The General Case

We now consider the general case of N sensors being present. As shown in Figure 2, sensors $1, 2, \cdots, N-1$ are assumed to be strategic while sensor N is assumed to be loyal. We denote the estimate $\hat{x}_N = \hat{x}_{r_N} = \hat{x}_l$ and $\Sigma_N = \Sigma_{r_N} = \Sigma_l$ and $e_N = e_l$ (subscript l stands for 'loyal'). There are two main difficulties that this general case brings. First, the strategic sensors can now compete against each other to gain the most compensation possible. This implies that we need to identify the *Nash equilibrium* behavior of the strategic sensors. Second, the central estimator needs to identify a way of verifying the information that each sensor transmits. While in the $N = 2$ case, it did not matter if the identity of the loyal sensor was known a priori to the central estimator; in the general N case, this assumption plays a pivotal role in designing a method to verify the information that each strategic sensor transmits. Accordingly, we can consider two cases

- The identity of the loyal sensor is known to the central estimator. In this case the i-th strategic sensor is offered a payment (following (5) in $N = 2$ case) as

$$p_i = c_i - \alpha_{1_i}\Sigma_{r_i} - \alpha_{2_i}(\hat{x}_l - \hat{x}_{r_i})^2. \quad (12)$$

This case can be addressed in a manner largely similar to the $N = 2$ case. We do not consider this case further in this paper.

- The identity of the loyal sensor is not known a priori to the central estimator. In this case, we assume that the central estimator pays the i-th strategic sensor according to the following function:

$$p_i = c_i - \alpha_{1_i}\Sigma_{r_i} - \alpha_{2_i}(\hat{x} - \hat{x}_{r_i})^2, \ \ \hat{x} = \frac{1}{N-1}\sum_{\substack{j=1 \\ j\neq i}}^{N} \hat{x}_{r_j}. \quad (13)$$

Accordingly, the utility of the i-th strategic is given by (13)

$$U_{s_i} = -h(a_i) - g\left(\Sigma_{r_i} - \Sigma_i, \hat{x}_{r_i} - \hat{x}_i\right) + p_{s_i}.$$

Design of the optimal contract: In this section, we study the interaction among the sensors and the central estimator under payment scheme (13). We first study the optimal reporting function for the strategic sensors. Then, we optimize the value of α_1 and α_2 for the central estimator to solve problem (4).

Theorem 5: Consider the problem stated in (4) for a general N case, with payment function (13). One Nash equilibrium is given by the strategy $\hat{x}_{r_i} = \beta_i \hat{x}_i$ for a priori chosen β_i, $\forall i = 1, \cdots, N$.

A particular case can be considered as follows.

Proposition 3: Consider the case when $\sigma_x^2 \to \infty$. Then, the proposed payment (13) by the central estimator induces 'truth telling' as a Nash equilibrium.
For this equilibrium, we present the optimal contract and the optimal effort and the global estimate it results in below.

Theorem 6: With the optimal contract proposed in (13), if $\sigma_x^2 \to \infty$, the following holds:

(i) $\Sigma_{r_i} = \Sigma_i - \alpha_{1_i}$

(ii) The optimal effort is given by $a_i^* = (\alpha_{1_i} + \alpha_{2_i})\frac{d\Sigma_i}{da_i}$ and it is an increasing function of α_{1_i} and α_{2_i}.

(iii) $\Sigma_g^{-1}\hat{x}_g = {\Sigma_l}^{-1}\hat{x}_l + \sum_{i=1}^{N-1}{\Sigma_i}^{-1}\hat{x}_i$, where $\Sigma_g^{-1} = {\Sigma_l}^{-1} + \sum_{i=1}^{N-1}{\Sigma_i}^{-1} - (N-1)\sigma_x^{-2}$

Please see [15] for the proofs of Theorems and Proposition 3.

Proposition 4: If $\sigma_x^2 \to \infty$ and given $\Sigma = \frac{\Sigma_l + \sum_{\substack{j=1 \\ j\neq i}}^{N-1}\Sigma_j}{(N-1)^2}$, the proposed contract structure in (13) satisfies

(i) $\alpha_{1_i} = 0$.

(ii) ex ante individual rationally constraint, if $c_i > {\alpha_{2_i}}^*(\Sigma + \Sigma_i(a_i^*)) - \frac{{a_i^*}^2}{2}$.

(iii) The optimal value of α_{2_i} is given by the solution of equation $\alpha_{2_i}\frac{d\Sigma_i}{d\alpha_{2_i}} = -(\Sigma + \Sigma_i)$. If $\alpha_{2_i}\frac{d^2\Sigma_i}{d\alpha_{2_i}^2} + 2\frac{d\Sigma_i}{d\alpha_{2_i}} < 0$, there exist a unique solution and the i-th strategic sensor improves his error covariance as Σ increases (as the quality of estimate provided by other sensors decreases).

Proof: Proof follows along the same lines as the proofs of Theorems 3 and 4 and Proposition 2 and is omitted for space constraints ■

IV. Conclusion and Future Work

In this paper, we designed an optimal contract between a central estimator and group of strategic sensors for a static centralized estimation problem. The central estimator incentivizes the sensors to invest resources to obtain measurements of high enough quality to be able to generate a local estimate with sufficient accuracy. However, since the effort invested and the estimate values are private information, a strategic sensor can misreport these quantities to obtain a higher payment. We proposed an optimal contract that minimizes the total payment by the central estimator by incentivizing the sensors both to make the maximal effort to improve the quality of estimation and not to misreport the estimate.

Future work will involve considering the dynamic estimation problem. Dynamic contract design is a significantly more involved problem. Also of interest would be considering contracts in the context of more general Bayesian or Dempster-Shafer fusion rules.

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