Decentralized Temperature Control via HVAC Systems in Energy Efficient Buildings: An Approximate Solution Procedure

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***Abstract*— This paper presents a real-time decentralized temperature control scheme via Heating Ventilation and Air Conditioning (HVAC) systems for energy efficient buildings, which balances user comfort and energy saving. Firstly, we introduce a thermal dynamic model of building systems and its approximation. Then a steady-state optimization problem is formulated, which aims to minimize the aggregate deviation between zone temperatures and their set points, as well as the building energy consumption. Because this problem is nonconvex, a convex relaxation approach is proposed, and the tightness of the relaxation is proved. We next develop a real-time decentralized algorithm to regulate the zone flow rate, so that the thermal dynamics can be driven to an equilibrium point which is the optimal solution of the steady-state optimization problem. Finally, a numerical example is proposed to illustrate the effectiveness of the control scheme.**

***Index Terms*— Temperature control, HVAC systems, convex relaxation, decentralized control, gradient algorithms.**

I. Introduction

According to an investigation by the United Nations, buildings are responsible for 40% of energy consumption, 70% of electricity consumption, and result in 30% of greenhouse gas emission [1]. Roughly speaking, Heating Ventilation and Air Conditioning (HVAC) systems in buildings account for half of the energy use [2]. It is therefore necessary to make them more energy efficient for environmental sustainability.

This paper aims to develop a real-time control scheme for HVAC systems in typical commercial buildings. More specifically, we aim to design a distributed/decentralized algorithm to guide each controllable component to properly adapt their behavior such that system-wide objectives are achieved under given operating conditions. The proposed control scheme: (i) is scalable with respect to building structures, (ii) respects the system operating constraints, (iii) ensures system efficiency, reliability and user comfort. Different from literature which often uses, e.g., model predictive control [3], [4], mean-field control [5], etc., the control scheme designed in this paper can be implemented in a much easier way.

The structure of this paper is as follows. In Section II, we provide the detailed problem setup, including an introduction of the HVAC system configuration, a thermal dynamic model and its approximation, and the objective optimization problem formulation. Since the objective optimization problem is nonconvex, in Section III, we present a convex relaxation approach. We show that this convex relaxation is always tight under a mild condition. Moreover, we develop a decentralized control scheme on real-time temperature regulation for the HVAC system, which drives the thermal dynamics to an equilibrium point that solves the objective optimization problem. In Section IV, a numerical example is given, using a building with four adjacent zones. Conclusions and future work are presented in Section V.

Notation: The positive projection of a function $h(y)$ on a variable $x \in [0, +\infty)$, $(h(y))^+_x$ is:

$$(h(y))^+_x = \begin{cases} h(y) & \text{if } x > 0 \\ \max(0, h(y)) & \text{if } x = 0 \end{cases} .$$

II. Problem Setup

A. HVAC system configuration

The schematic of a typical HVAC system is illustrated in Figure 1, which consists of an Air Handling Unit (AHU) for the whole building and a set of Variable Air Volume (VAV) boxes for each zone. The AHU is equipped with a damper, a cooling/heating coil, and a Variable Frequency Drive (VFD) fan: the damper mixes the return air from the building with the outside air; the cooling/heating coil cools down/heats up the mixed air; the VFD fan adjusts its own speed based on the total mass flow rate/opening controlled by VAV boxes to keep the duct pressure at a certain level, and drives the cooled/heated air to each zone. Each VAV box has a damper controlling the mass flow rate of the supply air, which is considered as the only controllable input to the system.

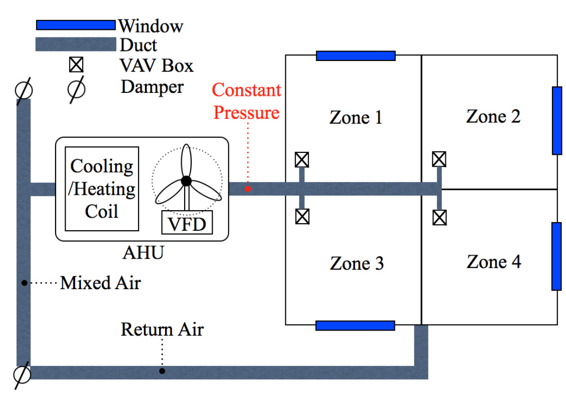

Fig. 1: Schematic of a typical AHU&VAV system.

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B. Thermal dynamic model and approximation

According to the above configuration, we model a given building as an undirected connected graph $(\mathcal{N}, \mathcal{E})$. Here $\mathcal{N}$ is the set of nodes representing zones/rooms, and $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ is the set of edges: if zones i and j are neighbors, then the edge (i, j) exists. Let $\mathcal{N}(i)$ denote the set of neighboring zones of zone i. The thermal dynamics for the building is described by a reduced Resistance-Capacitance (RC) model [6]:

$$C_i \dot{T}_i = \frac{T^o - T_i}{R_i} + \sum_{j \in \mathcal{N}(i)} \frac{T_j - T_i}{R_{ij}} + c_a m_i (T^s - T_i) + Q_i \qquad i \in \mathcal{N} \quad (1)$$

where C_i is the thermal capacitance, T_i is the indoor temperature, T^o is the outdoor temperature, R_i is the thermal resistance of the wall&window separating zone i and outside, R_{ij} is the thermal resistance of the wall separating zones i and j, c_a is the specific heat of the air, m_i is the flow rate of the supply air, T^s is the supply air temperature which is usually a constant [4], and $Q_i \geq 0$ is the heat gain from external sources. One property of (1) is that given T^o, m_i, Q_i, it is globally asymptotically stable. This can be directly obtained by rearranging (1) in state-space representation, and showing that the system matrix is Hurwitz.

In reality, due to the natural property of building materials, the thermal resistance of the wall separating zones, i.e., R_{ij}, is (sometimes much) larger than the thermal resistance of the wall&window separating zones and outside, i.e., R_i. Moreover, T_i, T_j of neighboring zones do not deviate too much from each other. These facts result in that the heat transfer between neighboring zones is not dominant. Therefore, we ignore the term $\sum_{j \in \mathcal{N}(i)} \frac{T_j - T_i}{R_{ij}}$ in system (1) (although this term is ignored in the design procedure, our controller can still behave well for the accurate model, as will be shown in Section IV) and obtain an approximate model:

$$C_i \dot{T}_i = \frac{T^o - T_i}{R_i} + c_a m_i (T^s - T_i) + Q_i, \; i \in \mathcal{N}. \quad (2)$$

Obviously, the global asymptotic stability property is preserved from model (1) to (2). Note that the flow rate m_i is considered as the only control input to each zone here.

C. The optimization problem

Each zone has a desired temperature which is the set point given by users. The objective is to regulate the temperature to be close to the set point in steady state, and to minimize the total energy consumption in the building. As a result, we consider the following steady-state optimization problem:

$$\min_{T_i, m_i} \sum_{i \in \mathcal{N}} \frac{1}{2} r_i (T_i - T_i^{set})^2 + \sum_{i \in \mathcal{N}} \frac{1}{2} s_i m_i^2 \quad (3a)$$

$$\text{s. t. } \frac{T^o - T_i}{R_i} + c_a m_i (T^s - T_i) + Q_i = 0 \quad (3b)$$

$$m_i^{min} \leq m_i \leq m_i^{max} \quad (3c)$$

$$\sum_{i \in \mathcal{N}} m_i \leq \overline{m} \quad (3d)$$

where $i \in \mathcal{N}$ in (3b)-(3c), r_i and s_i are positive weight coefficients, T_i^{set} is the temperature set point, $[m_i^{min}, m_i^{max}]$ is the range of m_i in which m_i^{min} is usually close to zero [3], and $\overline{m}$ is the upper bound of the total flow rate in the building. Note that T^o and Q_i are exogenous disturbances. We assume that (3) is feasible and satisfies Slater's condition [7]. For problem (3), we have four important remarks.

- In the objective function (3a), the term relating to the energy consumption, i.e., power consumptions of the supply fan and the cooling/heating coil, does not appear directly. Since the consumption of the supply fan is approximately proportional to the cube of the total supply air flow [4], and the consumption of the cooling/heating coil is an increasing function of the total air flow going through the coil, minimizing the energy consumption can be approximated by minimizing the total flow rate $\sum_{i \in \mathcal{N}} m_i$. Using a Cauchy-Schwarz inequality, we have $\sum_{i \in \mathcal{N}} \frac{1}{2} s_i m_i^2 \geq \frac{(\sum_{i \in \mathcal{N}} m_i)^2}{2 \sum_{i \in \mathcal{N}} \frac{1}{s_i}}$ which indicates that, rather than minimizing the consumption directly, the objective here aims to minimize its upper bound.
- Parameters r_i and s_i are determined by users: if users prefer more comfort, they can increase r_i and decrease s_i, and vice versa. Because of this flexibility, there is no need to impose constraints on the temperature comfortable range.
- In the cooling mode, $T^s \ll T_i^{set}, T^s \ll T_i, \forall i$ hold; in the heating mode, $T^s \gg T_i^{set}, T^s \gg T_i, \forall i$ hold. This is always true in practice. For example [4], in the cooling mode, $T^s = 12.8°\text{C}$ while $T_i^{set}, T_i \geq 21.5°\text{C}$.
- In constraint (3d), we do not impose any lower bound for the total flow rate as the lower bound usually equals $\sum_{i \in \mathcal{N}} m_i^{min}$. Also, $\overline{m} < \sum_{i \in \mathcal{N}} m_i^{max}$ holds, otherwise, this constraint would become redundant.

To conclude, the goal is to design m_i in a *decentralized* way so that (3) is solved when (2) reaches steady state.

III. An Approximate Solution Procedure

A. Convex relaxation

Problem (3) is nonconvex because the quadratic equality constraint (3b) is nonconvex in m_i, T_i. However, we can actually convexify it and show that the convexification is indeed exact. From (3b), we have

$$f_i(T_i) \triangleq \frac{\frac{T^o - T_i}{R_i} + Q_i}{c_a (T_i - T^s)} = m_i, \; i \in \mathcal{N}.$$

The physical meaning of $f_i(T_i)$ is the *approximate* flow rate of the supply air for zone i to stay at temperature T_i. Also, $f_i'(T_i) = \frac{\frac{T^s - T^o}{R_i} - Q_i}{c_a (T_i - T^s)^2}, \; f_i''(T_i) = \frac{-2(\frac{T^s - T^o}{R_i} - Q_i)}{c_a (T_i - T^s)^3} > 0, \; i \in \mathcal{N}$ hold, due to $T^s \ll T^o$ in the cooling mode, and $T^s \gg T^o$, $\frac{T^s - T^o}{R_i} > Q_i$ (as Q_i is usually less dominant, see, e.g., [3], [5]) in the heating mode. Since $f_i(T_i)$ is convex in T_i, we relax (3) as a convex optimization problem given by

$$\min_{T_i, m_i} \sum_{i \in \mathcal{N}} \frac{1}{2} r_i (T_i - T_i^{set})^2 + \sum_{i \in \mathcal{N}} \frac{1}{2} s_i m_i^2 \quad (4a)$$

$$\text{s. t. } \frac{\frac{T^o - T_i}{R_i} + Q_i}{c_a (T_i - T^s)} \leq m_i, \; i \in \mathcal{N} \quad (4b)$$

$$m_i^{min} \leq m_i \leq m_i^{max}, \ i \in \mathcal{N} \tag{4c}$$

$$\sum_{i \in \mathcal{N}} m_i \leq \overline{m}. \tag{4d}$$

B. Tightness of the convex relaxation

Before investigating the tightness of the convex relaxation, we make a related assumption.

Assumption 1. *In the cooling mode, $T_i^{set} < T^o, \forall i$ holds; in the heating mode, $T_i^{set} > T^o, \forall i$ holds. Moreover, the zone temperature set point satisfies $f_i(T_i^{set}) \geq m_i^{min}, \forall i$.*

This assumption is supported as follows. First, when the outdoor temperature is higher (in hot days)/lower (in cold days) than users' expectation (quantified as T_i^{set}), the HVAC system should be turned on. So we usually have $T_i^{set} < T^o, \forall i$ for cooling and $T_i^{set} > T^o, \forall i$ for heating. Note that $m_i^{min} \approx 0, \forall i$ holds as stated earlier, and $f_i(T_i)$ stands for the approximate flow rate for zone i to stay at temperature T_i. The inequality $f_i(T_i^{set}) \geq m_i^{min}$ means that in order to make the temperature of zone i be the set point, the flow rate needs to be no smaller than its minimum. Otherwise, we will have $f_i(T_i^{set}) < m_i^{min} \approx 0$ which can lead to $T_i^{set} \approx T^o$ or $T_i^{set} > T^o$ in the cooling mode, and $T_i^{set} \approx T^o$ or $T_i^{set} < T^o$ in the heating mode, a contradiction to the first part of this assumption. To sum up, this assumption can be regarded as a constraint for choosing the set point in each zone, which is completely decentralized.

The Lagrangian of problem (4) is given by

$$\begin{aligned} L = &\sum_{i\in\mathcal{N}} \frac{1}{2} r_i (T_i - T_i^{set})^2 + \sum_{i\in\mathcal{N}} \frac{1}{2} s_i m_i^2 + \lambda^+ \Big(\sum_{i\in\mathcal{N}} m_i - \overline{m} \Big) \\ &+ \sum_{i\in\mathcal{N}} \zeta_i \left(\frac{\frac{T^o - T_i}{R_i} + Q_i}{c_a (T_i - T^s)} - m_i \right) + \sum_{i\in\mathcal{N}} \mu_i^+ (m_i - m_i^{max}) \\ &+ \sum_{i\in\mathcal{N}} \mu_i^- (m_i^{min} - m_i) \end{aligned}$$

where $\zeta_i, \mu_i^+, \mu_i^-, \lambda^+$ are the Lagrange multipliers/dual variables for constraints (4b)-(4d). Since problem (4) is convex, feasible and satisfies Slater's constraint qualification, the Karush-Kuhn-Tucker (KKT) conditions are necessary and sufficient conditions for optimality [7], given by

$$\frac{\partial L}{\partial T_i} = r_i (T_i - T_i^{set}) + \zeta_i \frac{\frac{T^s - T^o}{R_i} - Q_i}{c_a (T_i - T^s)^2} = 0 \tag{5a}$$

$$\frac{\partial L}{\partial m_i} = s_i m_i - \zeta_i + \mu_i^+ - \mu_i^- + \lambda^+ = 0 \tag{5b}$$

$$\zeta_i (f_i(T_i) - m_i) = 0, \ \zeta_i \geq 0, f_i(T_i) - m_i \leq 0 \tag{5c}$$

$$\mu_i^+ (m_i - m_i^{max}) = 0, \ \mu_i^+ \geq 0, m_i - m_i^{max} \leq 0 \tag{5d}$$

$$\mu_i^- (m_i^{min} - m_i) = 0, \ \mu_i^- \geq 0, m_i^{min} - m_i \leq 0 \tag{5e}$$

$$\lambda^+ \left(\sum_{i\in\mathcal{N}} m_i - \overline{m} \right) = 0, \ \lambda^+ \geq 0, \sum_{i\in\mathcal{N}} m_i - \overline{m} \leq 0 \tag{5f}$$

where $i \in \mathcal{N}$ in (5a)-(5e).

Remark 1. *The convex relaxation from problem (3) to (4) is tight if and only if any solution of (5) satisfies $\zeta_i > 0$ or $\zeta_i = 0, f_i(T_i) = m_i, \forall i$.*

As for the tightness, we have the following theorem.

Theorem 1. *Under Assumption 1, the convex relaxation from problem (3) to (4) is always tight.*

Proof. Let us assume that there exists an i in the solution of (5) such that $\zeta_i = 0, f_i(T_i) < m_i$ hold. Then we can obtain $T_i = T_i^{set}$ from (5a), and $\mu_i^- = s_i m_i + \mu_i^+ + \lambda^+ > 0$ from (5b). This leads to $f_i(T_i^{set}) = f_i(T_i) < m_i = m_i^{min}$, a contradiction to Assumption 1. Based on Remark 1, the convex relaxation from problem (3) to (4) is always tight. □

C. A decentralized algorithm

Theorem 1 indicates that solving problem (3) is equivalent to solving problem (4), while (4) can be solved in either a centralized or distributed/decentralized way. Any centralized algorithm requires to measure the outdoor temperature T^o and the indoor heat gain Q_i in every zone. Because these exogenous disturbances can fluctuate in real-time and sometimes by a large amount, the cost of centralized algorithms would be expensive. In this section, we develop a real-time decentralized algorithm that does not need measurement of these exogenous disturbances.

Since (4) is a steady-state optimization problem and does not capture the transient behavior of model (2), to avoid confusion, we replace T_i with Z_i in (4) under which the optimal solution remains the same. Motivated by a standard primal-dual gradient method [8], [9], we come up with the following algorithm to solve (4):

$$\begin{aligned} \dot{Z}_i = &- k_{Z_i} \left(\frac{\partial L}{\partial T_i} \bigg|_{T_i = Z_i} \right) \\ = &k_{Z_i} \left(r_i (T_i^{set} - Z_i) - \zeta_i \frac{\frac{T^s - T^o}{R_i} - Q_i}{c_a (Z_i - T^s)^2} \right) \end{aligned} \tag{6a}$$

$$\dot{m}_i = - k_{m_i} \left(\frac{\partial L}{\partial m_i} \right) = k_{m_i} (-s_i m_i + \zeta_i - \mu_i^+ + \mu_i^- - \lambda^+) \tag{6b}$$

$$\dot{\zeta}_i = k_{\zeta_i} \left(\frac{\partial L}{\partial \zeta_i} \bigg|_{T_i = Z_i} \right)_{\zeta_i}^+ = k_{\zeta_i} \left(\frac{\frac{T^o - Z_i}{R_i} + Q_i}{c_a (Z_i - T^s)} - m_i \right)_{\zeta_i}^+ \tag{6c}$$

$$\dot{\mu}_i^+ = k_{\mu_i^+} \left(\frac{\partial L}{\partial \mu_i^+} \right)_{\mu_i^+}^+ = k_{\mu_i^+} (m_i - m_i^{max})_{\mu_i^+}^+ \tag{6d}$$

$$\dot{\mu}_i^- = k_{\mu_i^-} \left(\frac{\partial L}{\partial \mu_i^-} \right)_{\mu_i^-}^+ = k_{\mu_i^-} (m_i^{min} - m_i)_{\mu_i^-}^+ \tag{6e}$$

$$\dot{\lambda}^+ = k_{\lambda^+} \left(\frac{\partial L}{\partial \lambda^+} \right)_{\lambda^+}^+ = k_{\lambda^+} \left(\sum_{i\in\mathcal{N}} m_i - \overline{m} \right)_{\lambda^+}^+ \tag{6f}$$

where $i \in \mathcal{N}$ in (6a)-(6e), and $k_{Z_i}, k_{m_i}, k_{\zeta_i}, k_{\mu_i^+}, k_{\mu_i^-}, k_{\lambda^+}$ are positive scalars representing the controller gains. Note that T_i has its own dynamics given by (2) and thus can not be designed, which is why we replace T_i with Z_i, i.e., $Z_i, i \in \mathcal{N}$ are ancillary state variables. According to [8], it is true that (6) asymptotically converges to an equilibrium point which is the optimal solution of (4)/(3) by Theorem 1. Now using m_i in (6b) as the input to (2), we can naturally obtain a real-time decentralized algorithm to regulate (2) to a steady state which solves (3): under constant/slow-varying T^o, Q_i, the equilibrium point of the overall system (2) and (6) is unique, asymptotically stable (this is due to the cascade nature of the system, i.e., (6)→(2)), and T_i, m_i of the equilibrium point ($T_i = Z_i$) is the optimal solution of (3).

In Equations (6a) and (6c), the disturbances T^o, Q_i appear. Motivated by [10], to make the algorithm regardless of these terms, we substitute (2) into these equations to get

$$\dot{Z}_i = k_{Z_i}\left(r_i(T_i^{set} - Z_i) - \zeta_i \frac{\frac{T^s - T_i}{R_i} - C_i\dot{T}_i - c_a m_i(T_i - T^s)}{c_a(Z_i - T^s)^2} \right) \tag{7a}$$

$$\dot{\zeta}_i = k_{\zeta_i}\left(\frac{\frac{T_i - Z_i}{R_i} + C_i\dot{T}_i + c_a m_i(T_i - T^s)}{c_a(Z_i - T^s)} - m_i \right)^+_{\zeta_i} \tag{7b}$$

in which the derivative action $\dot{T}_i$ can be implemented by using a differentiator with some form of filtering [11]–[13].

The overall control algorithm (6b), (6d)-(6f) and (7) is completely decentralized and can be implemented as follow. Given $T^s, R_i, C_i, r_i, s_i, m_i^{min}, m_i^{max}$, each zone in the building collects T_i^{set} from users, locally measures its indoor temperature T_i, receives the feedback signal λ^+ from the supply fan/duct, and then uses these information to update $Z_i, m_i, \zeta_i, \mu_i^+, \mu_i^-$ based on (6b), (6d)-(6e) and (7). On the other hand, given $\overline{m}$, the supply fan/duct locally measures the total flow rate $\sum_{i\in\mathcal{N}} m_i$ (which is proportional to the fan speed [14]) and uses (6f) to update λ^+, then broadcast it. Here the parameters $T^s, R_i, C_i, m_i^{min}, m_i^{max}, \overline{m}$ are building specified and r_i, s_i are design parameters given by users.

Remark 2. *Other types of functions can be used to characterize the term relating to the energy consumption, i.e.,*

$$\min_{T_i, m_i} \sum_{i\in\mathcal{N}} \frac{1}{2} r_i (T_i - T_i^{set})^2 + \sum_{i\in\mathcal{N}} s_i g(m_i, T_i) \tag{8}$$

where $s_i > 0$, and the choice of $g(m_i, T_i)$ needs to ensure the convexity of the objective function. Due to space limitations, the result will be reported in a future paper.

IV. A Numerical Example

We now present the cooling case for a building with four zones as illustrated in Figure 1 (the heating case is not shown due to space limitations, but it is similar). The parameters of the overall system are randomly selected from [15] in which $\overline{m} = 0.4$kg/s, and profiles of the outdoor temperature and the indoor heat gain are shown in Figure 2. The simulation results are illustrated in Figures 3-4, in which curves labeled with "app" indicate the case of using approximate model (2) while the others indicate the case of using model (1). The total flow rate reaches its maximum from 10h to 18h, while does not saturate in other periods. We can see that there is always a tradeoff between comfort and energy saving, and the proposed controller behaves well for the accurate model.

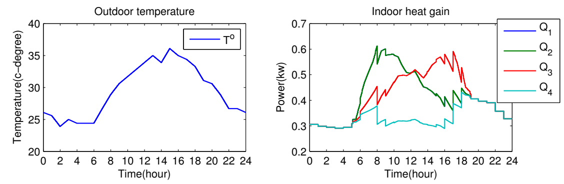

Fig. 2: Profiles of the disturbances ($Q_1 = Q_2$).

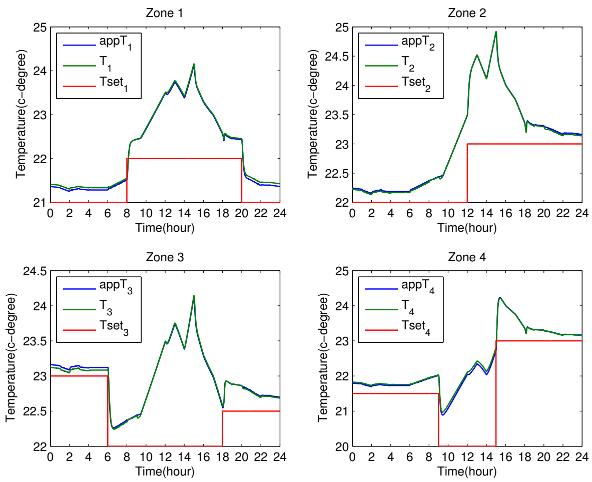

Fig. 3: Temperatures in each zone: curves labeled with "app" indicate the case of using the approximate model (2).

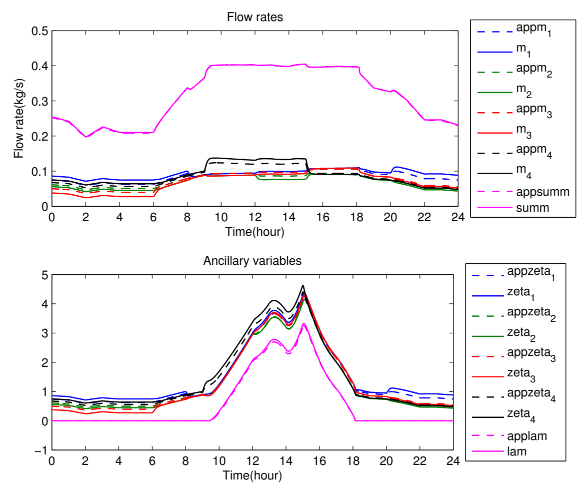

Fig. 4: Flow rates and ancillary variables.

V. Conclusion and Future Work

This paper presents a real-time decentralized temperature control scheme for HVAC systems of energy efficient buildings, which balances user comfort and energy saving. Although the control scheme is derived under an approximation, it behaves well, and its decentralized implementation is much easier than controllers proposed in literature.

Future research directions will include: revisiting the design under more accurate energy cost functions, as mentioned in Remark 2; developing distributed/decentralized algorithms under the accurate model (1); investigating, for example, the H_2 and H_∞ performances of the controlled system.

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