2017 American Control Conference
Sheraton Seattle Hotel
May 24–26, 2017, Seattle, USA

Mechanism Design for Reliability in Demand Response with Uncertainty

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Abstract— **In this paper, we consider a two-stage demand response (DR) program where a DR aggregator calls upon customers to reduce demands at stage II in order to clear a targeted amount of electricity supply deficit. At stage I, customers only possess distributions of their load shedding costs instead of accurate values because of the intrinsic uncertainty of load shedding. We design an outcome-contingent mechanism in which customers report their private types of the cost distributions and the DR aggregator selects customers based on the reported information. The mechanism guarantees incentive compatibility and individual rationality. In addition, the mechanism ensures high reliability of the DR. That is, the deviation between the total reduced loads and the targeted amount is small. We provide both theoretical analysis and numerical studies to demonstrate the high reliability.**

I. Introduction

Customer participation has been playing an important role in transforming the electricity grid into a more energy efficient and sustainable one. Consequently, demand response (DR), which uses incentives to change customer behavior of electricity usage, has attracted lots of discussion [1] [2] [3] [4] [5] [6], and real-world applications [7] [8] [9]. For instance, voluntary day-ahead DR is widely used in practice, e.g., the DR programs by PJM [9] and PGE [7]. In this setup, a DR aggregator announces an electricity deficit to customers and schedules a DR plan in the day ahead of DR. Customers join DR plan voluntarily for monetary credits (rewards) and yet are not obligated to do DR when the deficit happens, even if they have reported that they would respond.

The uncertainty of whether customers will commit to perform DR or not causes a serious reliability problem. That is, if there is a targeted amount of electricity deficit to clear, it is difficult to ensure a small deviation between the true total reduced demands and the targeted value. A large deviation will further cause difficulties in maintaining the supply-demand balance for the electricity grid [10], [11]. One way to increase customers' commitment may be through exerting a penalty of un-commitment on those who have reported that they would respond. However, a simple high penalty will fail because it may discourage customers from participating in the DR at the very beginning. But if the amount of penalty is carefully chosen, this might be avoided. The difficulty lies in how to choose the right penalty, especially due to the fact that the DR aggregator lacks private information from customers. This paper seeks a mechanism design (MD) approach for the DR aggregator to acquire information from customers and then choose the right customers and set the right amount of penalties in order to achieve high reliability, i.e., a small deviation.

Mechanism design (MD) has been used in many DR research projects [12] [13] [14] [15]. In MD, customers report their private information, such as cost of load shedding, to a DR aggregator; the DR aggregator selects DR customers and determines payment rules for the DR program. However, most of those day-ahead DR mechanisms neglect one important aspect of this problem – the uncertainty in customers' costs of load shedding [13] [15]. The uncertainty originates from collecting cost information for a future DR event. Customers, especially small ones, are usually not fully certain about their future costs of load shedding due to, for example, weather conditions or emergencies. Neglecting the cost uncertainty inevitably triggers strategic reporting, which results in defective estimation and control on customers' behaviors and thereby poor reliability. [14] is a previous attempt to deal with cost uncertainties. However, it only considers a specific case when customers costs follow binomial distribution.

To deal with the problems above, we design a mechanism for a two-stage DR program which involves customers with uncertain random costs of load shedding. The goal is to call upon customers to reduce demands in order to clear a targeted amount of electricity supply deficit. We assume that the targeted amount is M units and each customer is either able to shed one unit of load or not. At stage I, customers only possess distributions of their load shedding costs. The mechanism collects reports from customers for their cost distribution types. Based on the reported information, the mechanism chooses a subset of customers whom are asked to shed loads at stage II. Those customers receive a fixed reward but are also subject to a potential penalty if they do not perform DR at stage II. The selection rules and the penalties are designed in a way such that for each customer the best rational strategy is to report his true cost distribution type (*Incentive Compatibility*) and the expected net revenue of participating in such DR program is nonnegative (*Individual Rationality*). Moreover, we provide both theoretical analysis and numerical studies to demonstrate that our mechanism achieves high DR reliability. That is, the deviation between the total reduced demands and the targeted amount is very small. Lastly, we also numerically compare our mechanism with another two mechanisms to show the effectiveness of our MD in achieving high reliability.

The paper is organized as follows. In Section II, we introduce our DR setup and some basic MD theory. Section III presents the outcome-contingent mechanism and discusses the intuition behind it. In Section IV, we give an analytical upper bound for the deviation between the reduced demands and the targeted amount. In Section V, we introduce another two mechanisms for comparison purpose. Lastly, Section VI provides numerical studies. Due to space limit, we defer all the proofs to [16].

II. Problem formulation & Preliminaries

A. Problem setup

We consider a situation where there is a forecast supply deficit of electricity in the future and a DR aggregator would

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978-1-5090-5992-8/$31.00 ©2017 AACC

like to call upon customers to shed loads in order to clear a fixed quota of the deficit. In particular, we consider that the deficit quota is M units and each customer is able to shed one unit of load. The set of customers is denoted by $\mathcal{N} = \{1, \ldots, N\}$. Our demand response (DR) setup is motivated by the day-ahead DR experiments conducted by PGE and PJM [7] [9]. The demand response is operated at two stages:

- at stage I the DR aggregator chooses a subset of customers $J \subset \mathcal{N}$ and asks them to shed a unit of load at stage II;
- at stage II, each customer $i \in J$ decides whether to shed the unit of load or not, depending on his cost of load shedding, denoted by C_i.

A main challenge of the DR is the uncertainty of customer costs in load shedding. Before stage II, it is difficult, if not impossible, to know an exact value of the cost even for the customer himself. If the set of customers J is badly chosen and no one sheds the load at stage II, then the system will suffer a serious reliability issue, i.e., none of the deficit will be cleared. This motivates the work of this paper–to design a mechanism where the DR aggregator is able to select the right set of customers to minimize the deviation between the total reduced loads and the targeted level M.

Remark 1. *The deficit quota is usually different from the total deficit. It can be a fixed and deterministic value whereas the actual deficit is usually uncertain at stage I. In this paper, we consider a fixed deficit quota and the rest of the deficit can be cleared by other methods, such as reserve generators and load-tracking generators.*

Specifically, we model customer i's cost of load shedding as a nonnegative random variable C_i. We further parametrize the probability distribution of C_i by assuming that $\{C_i\}_{i \in N}$ independently follows a similar pattern, i.e., $C_i \sim F(\boldsymbol{\lambda}_i)$ where $F(\cdot)$ denotes a common distribution class and $\boldsymbol{\lambda}_i$ is an individual parameter vector for customer i. $\boldsymbol{\lambda}_i$ is assumed to be privately known to customer i. We call $\boldsymbol{\lambda}_i$ as the private type of customer i. Without causing any confusion, we will abuse notations by using $C_i(\boldsymbol{\lambda}_i)$ to denote the random variable C_i.

Remark 2. *The distribution class F can be very general and approximated using real data. Compared with reporting non-parametrized distributions, the DR aggregator can use the parameterized distribution $C_i \sim F(\boldsymbol{\lambda}_i)$ to reduce the complexity of a DR mechanism or auction.*

Example 1. *Suppose F stands for the uniform distribution. Agent i's cost distribution can be formulated as $\mu_i + U[-\sigma_i, \sigma_i]$ where $\mu_i \geq \sigma_i$. The private type of agent i is $\boldsymbol{\lambda}_i = (\mu_i, \sigma_i)$ which is the mean value and variance up to a scalar factor.*

The mechanism we consider in this paper adopts an outcome-contingent payment plan. At stage I, the aggregator requests each customer to report its private type $\boldsymbol{\lambda}_i$. Based on the reported information, the aggregator selects the group J of customers. A base reward w is given to everyone in the group J to incentivize customers to participate in the DR program. At stage II, customers will decide whether or not to shed a unit of load based on the realization of the cost C_i. We use a_i to denote the action of customer i: $a_i = 1$ means that customer i sheds a unit of load and $a_i = 0$ means otherwise. If customer i chooses not to shed the load, a penalty m_i will be exerted on the customer. The utility of customer i in the selected group J is given by,

$$u_i(a_i) = w - C_i a_i - m_i(1 - a_i).^1 \tag{1}$$

Remark 3. *The assumption $a_i \in \{0, 1\}$ can be satisfied by adopting certain payment rule: if the customer opts out during a DR event, the customer will not get partial credit. Similar rules have been used in the DR practice, e.g. [7].*

Each customer is assumed to be rational when they make decisions. Thus, at stage II, customer i will choose $a_i^* := \arg\max_{a_i \in \{0,1\}} u_i(a_i)$. Because of the randomness of C_i, a_i^* and $u_i(a_i^*)$ are random variables as well. Denote the expected utility as $\tilde{u}_i := \mathbb{E}_{C_i \sim F(\boldsymbol{\lambda}_i)} u_i(a_i^*)$. As it is a function of $\boldsymbol{\lambda}_i$, w, and m_i, we denote the expected utility as $\tilde{u}_i(\boldsymbol{\lambda}_i, w, m_i)$, which can be calculated using the formulation in the following lemma. In addition, for the ease of notation, we will use a_i to denote the a_i^* in the rest of the paper.

Lemma 1. *Given customer i's type $\boldsymbol{\lambda}_i$, the based reward w and a penalty m_i, the expected utility $\tilde{u}_i$ is given by*[2]

$$\tilde{u}_i(\boldsymbol{\lambda}_i, w, m_i) = w - \int_0^{m_i} Pr(C_i > t | \boldsymbol{\lambda}_i) dt \tag{2}$$

In our proposed mechanism, the base reward w is fixed and is the same for all customers in the selected group J. The fixed rate ensures a bounded total budget that is used to pay for the DR, and the same rate is used to ensure fairness. However, in order to incentivize customers to report truthful information $\boldsymbol{\lambda}_i$, individual m_i will be determined for each customer $i \in J$. The mechanism will be introduced in details in the next section. Here, we provide some discussion on the constraints when determining m_i. From the expression of $\tilde{u}_i$ in (2), the expected utility $\tilde{u}_i$ is a monotonically decreasing function on the penalty m_i. In order to incentivize customers to participate in the DR program, customers' expected utility should be guaranteed to be nonnegative. This is know as *individual rationality (IR)* in mechanism design (MD) literature. As a result, m_i should be lower than the maximum penalty $\tilde{m}_i$ for each $i \in J$,

$$\tilde{m}_i := \arg\max_{m_i} \{m_i | \tilde{u}(\boldsymbol{\lambda}_i, w, m_i) \geq 0\} \tag{3}$$

It is easy to check that for any $m_i \leq \tilde{m}_i$, the expected utility of customer i is nonnegative.

As discussed before, the main objective of this paper is to design a mechanism to ensure a small deviation between the total reduced loads and the targeted amount M units. The smaller the deviation is, the higher reliability the system has. Mathematically, we define the reliability index R as the following,

Definition 1. *The reliability index R is defined as the inverse of the deviation, i.e.,*

$$R^{-1} = \sqrt{\mathop{\mathbb{E}}_{\vec{C}, \vec{\boldsymbol{\lambda}}} (M - \sum_{i \in J} a_i)^2} \tag{4}$$

where $\vec{a} = \{a_1, \ldots, a_N\}$ and $\vec{\boldsymbol{\lambda}} = \{\boldsymbol{\lambda}_i, \ldots, \boldsymbol{\lambda}_N\}$.

[1] For simplicity, we assume risk neutrality, but our mechanism can be easily adapted to risk-averse or risk-seeking case.

[2] For discrete distribution, simply replace integrals with summations.

In this definition, the expectation is taken over the uncertain a_i and $\boldsymbol{\lambda}_i$. Here we assume that even though the exact customer type $\boldsymbol{\lambda}_i$ is private information, $\boldsymbol{\lambda}_i$ is drawn randomly from a publicly known distribution. This assumption is borrowed from Bayesian MD [17]. The particular form of the reliability in (4) is motivated by the standard deviation definition in statistics.

B. Preliminaries: introduction on mechanism design (MD)

Before our DR mechanism, here we briefly introduce the general MD.

Mechanism design considers a set of outcomes O, and a set of self-interested agents $\{1, \ldots, N\}$ each with a private cost function $c_i(o)$ where $o \in O$. A typical mechanism consists of three parts: 1) collecting bid b_i from each customer i, 2) a selection rule $x(\vec{b})$ selecting outcomes based on the bid profile $\vec{b} = \{b_1, \ldots, b_N\}$, 3) a payment rule $p_i(\vec{b}, x(\vec{b}))$ based on reports and the selection rule[3]. Agents are assumed to be rational and bid to maximize their utilities $u_i(\vec{b}, c_i)$. A bidding strategy s_i specifies a mapping from private cost c_i to a bid b_i.

Definition 2. *A bidding strategy profile* $(s_1^*, \ldots, s_N^*)$ *is called a dominant strategy equilibrium (DSE) if*

$$u_i(s_i^*(c_i), s_{-i}(c_{-i}), c_i) \geq u_i(s_i(c_i), s_{-i}(c_{-i}), c_i)$$

for any i, s_i, s_{-i}, c_i and c_{-i}. Here the index $-i$ means all agents excluding i.

A mechanism collecting private cost C_i directly is called a *direct-revelation mechanism (DRM)*. One major goal of MD is to elicit truthful private information from customers, which is strictly defined as follows:

Definition 3. *A DRM is incentive-compatible (IC) if truthful reporting strategy $s_i(c_i) = c_i$ is a DSE.*

Besides, a mechanism should guarantee *individual rationality (IR)* for voluntary participation.

Definition 4. *A DSE is IR if it guarantees nonnegative utilities.*

Without loss of generality, we will focus on designing an IC and IR DRM, due to the following theorem.

Theorem 1. *(Revelation Principle [17]): Any mechanism with a DSE can be implemented as an IC DRM.*

III. Mechanism

In this section, we first discuss the intuition behind our mechanism regarding the reliability and IR property. We then present our mechanism and discuss more formally why our mechanism is IC and IR and how it achieves high reliability.

For the reliability analysis, we split the squared deviation, which is $(R^{-1})^2$, into two parts Error1 and Error2:

$$\begin{aligned}
\left(R^{-1}\right)^2 &= \mathop{\mathbb{E}}_{\vec{\boldsymbol{\lambda}}}(M - \sum_{i\in J} Pr(a_i = 1|\vec{\boldsymbol{\lambda}}))^2 \\
&+ \mathop{\mathbb{E}}_{\vec{\boldsymbol{\lambda}}} \sum_{i\in J} Pr(a_i = 1|\vec{\boldsymbol{\lambda}})(1 - Pr(a_i = 1|\vec{\boldsymbol{\lambda}})) \\
&= \underbrace{\mathop{\mathbb{E}}_{\vec{\boldsymbol{\lambda}}}(M - \sum_{i\in J} Pr(C_i \leq m_i|\vec{\boldsymbol{\lambda}}))^2}_{\text{Error1}} \\
&+ \underbrace{\mathop{\mathbb{E}}_{\vec{\boldsymbol{\lambda}}} \sum_{i\in J} Pr(C_i \leq m_i|\vec{\boldsymbol{\lambda}})(1 - Pr(C_i \leq m_i|\vec{\boldsymbol{\lambda}}))}_{\text{Error2}}
\end{aligned}$$

To achieve high reliability, both Error1 and Error2 should be kept small.

For small Error1, the expectation of the total reduced loads $\sum_{i\in J} Pr(C_i \leq m_i|\vec{\boldsymbol{\lambda}})$ should be close to M. This calls for eliciting private cost distribution from customers since $Pr(C_i \leq m_i|\vec{\boldsymbol{\lambda}})$ is private information thus unavailable to the DR aggregator. In addition to a good estimation of $Pr(C_i \leq m_i|\vec{\boldsymbol{\lambda}})$, a mechanism should ensure the estimated $\sum_{i\in J} Pr(C_i \leq m_i|\vec{\boldsymbol{\lambda}})$ be close to M.

Error2 relies on the number of customers in J and their probability of load shedding $Pr(C_i \leq m_i|\vec{\boldsymbol{\lambda}})$. A mechanism keeps small Error2 by i) selecting fewer customers and ii) incentivizing customers to respond with a higher probability. Note that i) conflicts with the requirement in Error1 since a few customers may not be sufficient to make $\sum_{i\in J} Pr(C_i \leq m_i|\vec{\boldsymbol{\lambda}})$ close to M. Hence a tradeoff should be made between Error1 and Error2. As for ii), one way to incentivize customers to respond is to have high penalties. However, if penalties are set too high, the MD will not guarantee IR. As discussed in Section II, IR requires m_i to be no greater than the maximum penalty $\tilde{m}_i$ for each i. This shows that there is a trade-off between reliability and IR as well.

A. Mechanism

We are ready to present our mechanism below. At stage I, the DR aggregator will perform the following procedure. (Ties are broken randomly.)

- (Bids) Ask everyone to report their types. Customer i's report, denoted as $\boldsymbol{\lambda}_i'$, may not be truthful for all i. The report profile is denoted by $\vec{\boldsymbol{\lambda}}'$.
- (Selection rule) Relabel customers by a decreasing order of $\tilde{m}_i$: $\tilde{m}_1 \geq \cdots \geq \tilde{m}_n$. J consists of the first k agents where k is defined as

$$\arg\min_j \{j| \sum_{i=1}^{j} Pr(C_i \leq m_i'|\vec{\boldsymbol{\lambda}}') \geq M - \beta, m_i' = \tilde{m}_j\} \tag{5}$$

  $\beta = 1/2$ with reasons provided in Section III-B.
- (Payment rule) If agent $i \in J$, its payment involves two parts: w and m_i, where w is a fixed reward and m_i is determined by the following rule. Excluding i, do the same selection rule again and find the index $k'(i)$ of the last agent selected. The penalty of agent i is $m_i = \tilde{m}_{k'(i)}$

When the selection rule terminates without a k, let $k = N$ and penalties be 0. When N is not enough to determine m_i, let $m_i = \tilde{m}_N$. Section VI will show gathering a few customers is sufficient to avoid these cases. Therefore, in the following analysis, we assume that customers are enough for implementing the selection and payment rules.

B. Discussion: Properties of our mechanism

Firstly, the following theorem proves that our mechanism is both IC and IR. Hence, customers will voluntarily join the DR and report their true distribution types.

Theorem 2. *The mechanism described above is IC and IR.*

[3]Strictly speaking, this is called as a mechanism with money.

Because the mechanism is IC, in the rest of the paper, we will let $\boldsymbol{\lambda}' = \boldsymbol{\lambda}$. Next we discuss why the mechanism is able to achieve high reliability by following the discussion at the beginning of this section.

Error1 is ensured to be small mainly because of the selection rule. Firstly, we note that if the penalty m_i is chosen to be $\tilde{m}_k$ as in (5), then the expected total reduced loads $\sum_{i=1}^{k} Pr(C_i \leq m_i|\vec{\boldsymbol{\lambda}})$ is guaranteed to be within the interval $[M-\beta, M+(1-\beta)]$ where β is chosen to be $1/2$ to minimize $\max(1-\beta, \beta)$. Though in order to ensure the IC property, the penalty m_i is chosen to $\tilde{m}_{k'(i)}$ in the payment rule, $\tilde{m}_k$ and $\tilde{m}_{k'(i)}$ are not very different from each other when the number of customers N is relatively large. This is formally proven in Section IV of [16] and further numerically verified in Section VI of [16]. As a result, the expected total reduced demands $\sum_{i=1}^{k} Pr(C_i \leq m_i|\vec{\boldsymbol{\lambda}})$ is close to M, meaning that Error1 is small.

For Error2, the selection rule achieves a good tradeoff between Error1 and Error2 by selecting the least number of customers to ensure small Error1, and the payment rule achieves a good tradeoff between a small Error2 and IR by selecting a high enough penalty without exceeding $\tilde{m}_i$.

Remark 4. *Roughly speaking, maximum penalty reflects the customers' probability of load shedding, but cannot perfectly determine all customers with the highest probability due to multi-dimensional type space. This turns out to have little effect on reliability especially with a large number of customers as shown in Section IV and VI*

IV. Theoretical Guarantee on Reliability

This section provides an analytical upper bound for the deviation. To obtain the bound, we make the following assumptions and introduce a few useful definitions and lemmas.

A. Main assumptions

The first assumption is on the distribution of private types.

Assumption 1. *Agents' types $\{\boldsymbol{\lambda}_i\}_{i\in N}$ are independent and identically distributed (i.i.d.) with a continuous distribution H. The support set of H, denoted by T, is compact.*

Assumption 1 ensures that customer maximum penalties $\tilde{m}_i$ are i.i.d.. As a result, $\tilde{m}_1, \ldots, \tilde{m}_N$ with new labels are order statistics where $\tilde{m}_k$ denotes the kth largest maximum penalty. In the rest of this section, we will omit subscript i in the notations as customers are i.i.d, except when dealing with order statistics. For example, we will use $\tilde{m}(\boldsymbol{\lambda})$ to denote the maximum penalty for a customer with a random type $\boldsymbol{\lambda}$.

Next is the continuity assumption.[4].

Assumption 2. *$Pr(C(\boldsymbol{\lambda}) \leq m|\boldsymbol{\lambda})$ is continuous with respect to $(m, \boldsymbol{\lambda})$, and $\tilde{m}(\boldsymbol{\lambda})$ is continuous with $\boldsymbol{\lambda}$.*

Assumption 1 and 2 imply that $\tilde{m}$ is bounded. The maximum of $\tilde{m}$ is denoted as $\bar{m}$.

We also assume at least some customers are likely to perform DR at stage 2 in Assumption 3. This can be guaranteed by having a large enough base reward. The requirement on w is given in Lemma 2.

Assumption 3. $\max_{\boldsymbol{\lambda}\in T, m\leq \tilde{m}(\boldsymbol{\lambda})} Pr(C(\boldsymbol{\lambda}) \leq m|\boldsymbol{\lambda}) > 0$

[4]The continuity assumption can be generalized to assuming discontinuity of the first kind. The bound is more complex but proof ideas are the same.

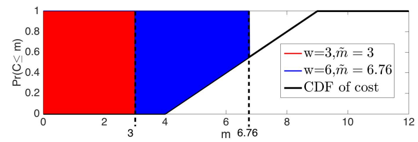


Fig. 1: $Pr(C < \tilde{m}(\boldsymbol{\lambda})|\boldsymbol{\lambda})$ with different w

Lemma 2. *$\bar{m} > w$ is equivalent to Assumption 3*

Although it seems that $\bar{m} > w$ requires w to be small enough, it is actually the opposite way because $\bar{m}$ implicitly depends on w and this inequality only holds when w is large enough. The following example helps explain this counter-intuitive statement.

Example 2. *Consider $C \sim U[4, 9]$. Figure 1 shows the relation between w and $\tilde{m}$ together with $Pr(C \leq \tilde{m}|\boldsymbol{\lambda})$.*[5] *When $w = 3$, $\tilde{m} = 3$ and $Pr(C \leq \tilde{m}|\boldsymbol{\lambda}) = 0$. If w is increased to 6, $\tilde{m} \approx 6.76$, and $Pr(C \leq \tilde{m}|\boldsymbol{\lambda}) \approx 0.55 > 0$. The critical value of w is 4. When $w > 4$, we have $\tilde{m} > w$ and $Pr(C \leq \tilde{m}|\boldsymbol{\lambda}) > 0$.*

Note that Assumptions 1-3 are satisfied by many distribution classes, e.g., exponential distribution and uniform distribution. We use these two distribution classes as examples to illustrate the assumptions.

Example 3. *Consider F as exponential distribution. $C_i = \mu_i + \sigma_i X_i$ where $\boldsymbol{\lambda}_i = (\mu_i, \sigma_i)$, mean value $\mu_i \sim U[15, 20]$, variance $\sigma \sim U[5, 10]$ and X_i i.i.d. from $Exp(1) - 1$. The base reward $w = 14$.*

$$\tilde{m}_i(\boldsymbol{\lambda}_i) = \begin{cases} 14, & \mu_i - \sigma_i \geq 14 \\ \mu_i - \sigma_i + \sigma_i \log(\frac{\sigma_i}{\mu_i - 14}), & \mu_i - \sigma_i < 14 \end{cases}$$

$$Pr(C \leq m|\boldsymbol{\lambda}) = \begin{cases} 1 - \exp(-1 + \frac{\mu - m}{\sigma}), & m \geq \mu - \sigma \\ 0, & m \leq \mu - \sigma \end{cases}$$

$$\bar{m} = 5 + 10\log(10) > w$$

Thus Assumption 2 and 3 are satisfied.

Example 4. *Consider F to be uniform distribution. $C_i = \mu_i + \sigma_i X_i$ where $\boldsymbol{\lambda}_i = (\mu_i, \sigma_i)$, mean value $\mu_i \sim U[16, 20]$, variance (up to a factor) $\sigma \sim U[12, 16]$ and X_i i.i.d. from $U[-1, 1]$. The base reward $w = 15$.*

$$\tilde{m}(\boldsymbol{\lambda}) = \begin{cases} 15, & \mu - \sigma \geq 15 \\ \mu + \sigma - 2\sqrt{\sigma}\sqrt{\mu - 15}, & \mu - \sigma < 15 \end{cases}$$

$$Pr(C \leq m|\boldsymbol{\lambda}) = \begin{cases} 0, & m < \mu - \sigma \\ \frac{m-\mu}{2\sigma} + 1/2, & \mu - \sigma \leq m \leq \mu + \sigma \\ 1, & m > \mu + \sigma \end{cases}$$

$$\bar{m} = 24 > w$$

Thus Assumption 2 and 3 are satisfied.

B. The Deviation Upper Bound

Next is the deviation upper bound in Theorem 3.

Theorem 3. *Given Assumption 1-3, the mechanism stated in Section III achieves the following reliability:*

$$R^{-1} \leq \sqrt{\eta_\epsilon + [N/4 + M^2 - \eta_\epsilon]Pr(A_\epsilon)},\ \forall 0 < \epsilon < \bar{m} - w \tag{6}$$

[5]Here we only have one type of $\boldsymbol{\lambda}$, i.e., the cardinality $|T| = 1$.

where

$$\eta_\epsilon = K_\epsilon \xi(\bar{p}^{\bar{m}-\epsilon}, \underline{p}^{\bar{m}-\epsilon}) + (K_\epsilon d(\epsilon) + 1/2)^2$$

$$\underline{p}^{\bar{m}-\epsilon} = \min_{\boldsymbol{\lambda} \in \Gamma(\epsilon)} Pr(C \leq \bar{m} - \epsilon | \boldsymbol{\lambda}) > 0, \forall 0 < \epsilon < \bar{m} - w$$

$$\bar{p}^{\bar{m}-\epsilon} = \max_{\boldsymbol{\lambda} \in \Gamma(\epsilon)} Pr(C \leq \bar{m} | \boldsymbol{\lambda})$$

$$d(\epsilon) = \max_{\boldsymbol{\lambda} \in \Gamma(\epsilon)} Pr(C \leq \bar{m} | \boldsymbol{\lambda}) - Pr(C \leq \bar{m} - \epsilon | \boldsymbol{\lambda})$$

$$\Gamma(\epsilon) = \{\boldsymbol{\lambda} \in T | \tilde{m}(\boldsymbol{\lambda}) \geq \bar{m} - \epsilon\}$$

$$K_\epsilon = \lceil \frac{M - 1/2}{\underline{p}^{\bar{m}-\epsilon}} \rceil, \; A_\epsilon = \{\tilde{m}_{K_\epsilon + 1} < \bar{m} - \epsilon\}$$

$$Pr(A_\epsilon) = G(\bar{m} - \epsilon)^N \sum_{i=0}^{K_\epsilon} \binom{N}{i} \left(\frac{1 - G(\bar{m} - \epsilon)}{G(\bar{m} - \epsilon)} \right)^i$$

$$G(\bar{m} - \epsilon) = Pr(\tilde{m}(\boldsymbol{\lambda}) < \bar{m} - \epsilon)$$

$$\xi(\bar{p}^{\bar{m}-\epsilon}, \underline{p}^{\bar{m}-\epsilon}) = \begin{cases} \underline{p}^{\bar{m}-\epsilon}(1 - \underline{p}^{\bar{m}-\epsilon}) & \textit{If } \frac{1}{2} < \underline{p}^{\bar{m}-\epsilon} \\ \bar{p}^{\bar{m}-\epsilon}(1 - \bar{p}^{\bar{m}-\epsilon}) & \textit{If } \bar{p}^{\bar{m}-\epsilon} < \frac{1}{2} \\ 1/4, & \textit{otherwise} \end{cases}$$

C. Effect of N on the Upper Bound in (6)

We claim that the upper bound (6) is of order $O(\sqrt{M}) + O(\sqrt{Poly(N,M)Pr(A_\epsilon)})$ where the second part quickly goes to zero with large N. To show this, we first note that η_ϵ is independent of N and can be bounded by $O(M)$ with a small enough ϵ by Lemma 3.

Lemma 3. $\lim_{\epsilon \to 0} d(\epsilon) = 0$. *Therefore, there exists $\epsilon > 0$ such that $\eta_\epsilon \sim O(M)$.*

The rest of bound (6) is the square root of $[N/4 + M^2 - \eta_\epsilon]Pr(A_\epsilon)$ which is a polynomial of N and M multiplied by $Pr(A_\epsilon)$. The following lemma shows that this part vanishes to zero when N goes to infinity.

Lemma 4. *When $\epsilon > 0$, for any finite-degree $Poly(N,M)$,*

$$\lim_{N \to \infty} Poly(N,M)Pr(A_\epsilon) = 0 \tag{7}$$

Thus, (6) is $O(\sqrt{M}) + O(\sqrt{Poly(N,M)Pr(A_\epsilon)})$, and is approximately $O(\sqrt{M})$ when N goes to infinity, as shown in Corollary 1.

Corollary 1. *When the number of agents goes to infinity:*

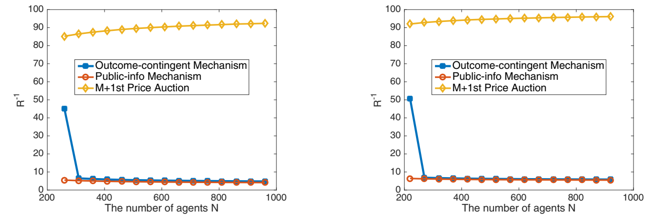
$$\lim_{N \to +\infty} R^{-1} \leq \sqrt{(\frac{M - 1/2}{\underline{p}^{\bar{m}}} + 1)\xi(\bar{p}^{\bar{m}}, \underline{p}^{\bar{m}}) + 1/4} \tag{8}$$

Although the analysis on N is only about the bound, simulation results in Section VI will show that R^{-1} itself enjoys the similar property.

Remark 5. *Notice that here we keep ϵ in the bound* (6) *instead of assigning a small enough value to ϵ. This is because when N is not very large, selecting a larger ϵ may provide a better bound.*

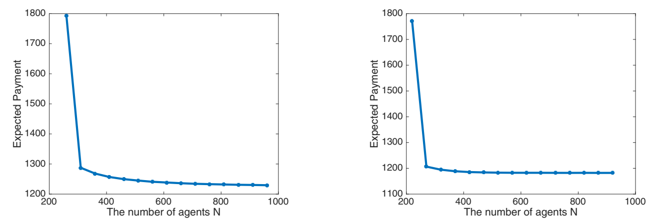
V. Other Mechanisms for Comparison

For the sake of comparison, we introduces two other mechanisms: public-info mechanism and $(M+1)$st price auction. Public-info mechanism provides a near-optimal reliability by unrealistically assuming the customers' cost distribution are public knowledge. It will serve as a benchmark and demonstrates the reliability loss due to private costs. $(M+1)$st price



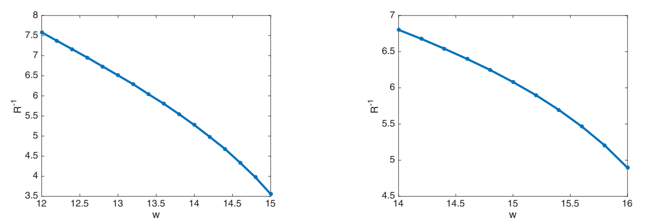
(a) Exponential Costs (b) Uniform Costs

Fig. 2: The relation between N and R^{-1} of our mechanism with private information, and R^{-1} of public-info mechanism



(a) Exponential Costs (b) Uniform Costs

Fig. 3: The relation between N and the total payment



(a) Exponential Costs (b) Uniform Costs

Fig. 4: The relation between the reward w and R^{-1}

auction is a commonly used mechanism in determinant-cost scenarios. Applying this to a system with random costs will show the consequence of neglecting cost uncertainties. The numerical comparisons are provided in SectionVI.

A. Public-info mechanism

Assuming public cost information, the DR aggregator can calculate customer i's the maximum penalty $\tilde{m}_i$ and probability of load shedding with the maximum penalty $Pr(C_i \leq \tilde{m}_i | \boldsymbol{\lambda}_i)$ for all i. Then the DR aggregator relabels customers in a decreasing order of $Pr(C_i \leq \tilde{m}_i | \boldsymbol{\lambda}_i)$ and

- (Selection rule): selects first k customers such that the total expected load reduction reaches $M - 1/2$ for the first time, (let $k = N$ if k is not found before termination)
- (Payment rule): for each selected customer i, offers the base reward w and the maximum penalty $\tilde{m}_i$ if he does not perform DR at stage II.

Intuitively speaking, the maximum penalty forces all customers to perform load shedding to the largest extent without violating IR, and the allocation rule priorly selects customers with the highest probabilities of response in order to ensure high reliability. Thus the public-info mechanism provides a near-optimal DR plan, which is unrealistic to implement in practice due to private costs.

B. $(M+1)$st price auction

Most of the current DR plans assume determinant costs, which is analogous to the classic multi-item auction scenario where N customers bid for M units. We apply the $(M+1)$st price auction (a generalization of second-priced auction) to this setting. In this auction, the DR aggregator collects customer cost values and re-indexed customers by an increasing order of reports. (Ties are broken randomly and $N > M$ is assumed for simplicity.)

- (Selection rule): Select the first M customers.
- (Payment rule): Provide each selected customer i with a payment equal to the report of the $(M+1)$st customer if customer i reduces one unit of load, and zero otherwise.

In a world without random costs, this auction is IC and IR [18], and provides good reliability since all M customers will shed loads at stage II due to utility maximization. However, when this auction is applied to reality, cost uncertainties will result in the following equilibrium:

Theorem 4. *It is a DSE when each customer i bids $\underline{C}_i$, where $\underline{C}_i := \max\{c | Pr(C_i \geq c) = 1\}$.*

The theorem shows that customers tend to report their minimal possible cost in order to be selected, leading to a low penalty as well. As a result, there are not enough incentives for customers to shed loads, which leads to poor reliability as shown in Section VI.

VI. Numerical Results

This section provides numerical results showing the high reliability achieved by our mechanism and the comparison with the public-info mechanism and the $(M+1)$st price auction introduced in the previous section. Additional discussion on budget is also included.

We consider two distribution classes of customers' costs: exponential distribution and uniform distribution. Most parameters are given in Example 3 and 4. For each class, the targeted DR value M is 100 units. Each scenario is iterated for 800 times to approximate the expectation in reliability definition. The simulation results are similar for the two distribution cases, so the discussion will only focus on the exponential case.

A. Reliability Analysis and Comparison with Public-Info mechanism and $(M+1)$st Price Auction

Figure 2 shows that when $N = 260$, R^{-1} of our mechanism is large. This is because customers are insufficient when N is small. When $N \geq 310$, there are sufficient customers and R^{-1} of our mechanism drops to a low level (less than 7), which means our mechanism provides good reliability as long as customers are enough. In addition, the ratio of sufficient N to M is less than 4 in this case, which is a relatively small number to guarantee high reliability. Furthermore, we see R^{-1} decreases with N, so recruiting more customers improves reliability. Moreover, when N is sufficient, reliability provided by our mechanism and public-info mechanism are almost the same, indicating an almost negligible reliability loss caused by private cost in our designed mechanism. Lastly, the $(M+1)$st price auction performs poorly when applied to the DR with random costs, which further confirms the importance of considering random costs when designing DR mechanisms.

B. Discussion on Budget

Payments are provided as participation incentives for customers. If the total payment is too high, it may be not worthwhile to clear the supply deficit through DR programs. Thus here we discuss the budget used in our proposed mechanism. Figure 3 shows that the payment remains at a low level when there are enough customers.

Given more budget, the base reward can be increased and the reliability will be improved accordingly as shown in Figure 4, where $N = 600$, and w is from 12 to 15 for exponential costs, and from 14 to 16 for uniform costs .

VII. Conclusion

This paper presents a two-stage DR mechanism to handle cost uncertainties of self-interested customers. Reliability is defined as the inverse of deviation between the total reduced loads and the targeted value. Our mechanism achieves high reliability as well as being IC and IR. Moreover, the reliability can be improved by recruiting more customers and/or increasing the budget when adopting our mechanism. Future work includes investigating other possible mechanisms and generalizing the assumptions on the cost uncertainties.

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