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Real Time Economic Dispatch for Power Networks: A Distributed Economic Model Predictive Control Approach

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***Abstract*— Fast power fluctuations pose increasing challenges on the existing control structure for power networks. One challenge is how to incorporate economic performance and constraint satisfaction in the operation. Current state of the art controllers are based on online steady-state optimization algorithms, which guarantee optimal *steady-state* performance. A natural extension of this trend is to consider economic model predictive control (EMPC), a dynamic optimization method, which can give guarantees on transient economic performance and constraint satisfaction. We show that the real time economic dispatch problem can be posed as an EMPC problem and provide corresponding transient guarantees for feasibility, stability and economic performance. Next, we show how the corresponding optimization problem can be solved online with dual distributed optimization and study stopping conditions due to real time requirements. This leads to an inexact solution of the optimization problem and we provide guarantees for this inexact distributed EMPC. Finally, we present simulation results showing constraint satisfaction and superior economic performance of the EMPC approach compared to state of the art solutions.**

I. INTRODUCTION

The standard control structure for power networks consists of different controllers that operate at different time scales with different control goals [1]. The fastest reaction to disturbances comes from primary frequency control, which absorbs load changes by adjusting the power within seconds. On a secondary level, the resulting frequency deviation is regulated by automatic generation control (AGC) and the nominal frequency is restored. On the slowest time scale, a centralized economic dispatch (ED) problem is solved to determine the economically optimal operating point, that satisfies the system constraints.

Increasing renewable and distributed energy generation leads to low system inertia and fast, large fluctuations in the power networks. This can lead to line flow constraint violations, large frequency deviations and a general deterioration of the economic performance. To address these problems, different control structures have been introduced in [2], [3], [4]. Most of these approaches pose a simplified economic dispatch as a steady-state optimization problem and implement the corresponding primal-dual algorithm as real time controller for the system. If the uncontrolled system is stable, the resulting closed-loop system converges to the optimal steady-state. Correspondingly, these controllers can only provide guarantees for *steady-state* operation, which is is hard to attain due to time-varying fluctuations. Constraint satisfaction and economic efficiency in transient operation is of paramount importance, but cannot be guaranteed by the above mentioned algorithms.

Model predictive control (MPC) is a well established control method, that can guarantee performance during transient operation including constraint satisfaction [5]. Recently, the extension of MPC to large scale (distributed) systems and direct consideration of general (economic) criteria has gained much research interest. In this paper, we propose to use distributed economic model predictive control (DEMPC) [6] with inexact minimization to solve the real time economic dispatch problem. Due to real time requirements and limited computational resources, it is usually not feasible to obtain an exact solution to the MPC optimization problem in each time step, which results in an inexact minimization resulting from finite dual iterations. We provide theoretical guarantees on constraint satisfaction, stability and economic performance for DEMPC, that only requires an inexact solution to the underlying optimization problem. Thus, this control approach is realistic for large scale systems and fast dynamics with limited computation and communication resources. The control algorithm guarantees transient constraint satisfaction and typically shows superior economic performance compared to the state of the art solutions. This is also in contrast to previous MPC approaches [7], [8], which neither considered economic performance nor hard state constraint satisfaction under inexact distributed optimization.

The paper is structured as follows: Section II presents the network model and describes the economic dispatch problem. Section III formulates the problem as a DEMPC problem and presents corresponding theoretical guarantees. Section IV addresses practical issues concerning the online optimization with real time requirements and provides modifications for DEMPC with inexact minimization with sufficient conditions for constraint satisfaction. Section V compares the DEMPC with a state of the art controller in simulation scenarios. Section VI concludes the paper and discusses further research questions.

A. Notation

The real numbers are denoted by $\mathbb{R}$ and the natural numbers are $\mathbb{N}$. The identity matrix is denoted by $I_n \in \mathbb{R}^{n\times n}$.

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For discrete time dynamics we use standard notation x^+, to denote the variables x at the next time step.

II. Economic Dispatch for Power Networks

A. Network Model

A power network can be modeled as an undirected graph $(\mathcal{N}, \mathcal{E})$, where each node $i \in \mathcal{N}$ represents a bus and the edges $(i,j) \in \mathcal{E} \subset \mathcal{N} \times \mathcal{N}$ represent the physical coupling with the line power flow P_{ij}. The buses are separated into generators $\mathcal{G}$ and loads $\mathcal{L}$ with $\mathcal{N} = \mathcal{G} \cup \mathcal{L}$. The neighbors of subsystems i are given by $\overline{\mathcal{N}}_i = \{j \in \mathcal{N} | (i,j) \in \mathcal{E}\}$ and the full neighborhood including subsystem i is given by $\mathcal{N}_i = \overline{\mathcal{N}}_i \cup \{i\}$. The overall state $x \in \mathbb{R}^n$ consists of local states $x_i \in \mathbb{R}^{n_i}$ of subsystem i with $x = [x_1^\top, \ldots, x_{|\mathcal{N}|}^\top]^\top$, $n = \sum_{i \in \mathcal{N}} n_i$. The states of the full neighborhood $\mathcal{N}_i$ are denoted by $x_{\mathcal{N}_i} \in \mathbb{R}^{n_{\mathcal{N}_i}}$ with $n_{\mathcal{N}_i} = \sum_{j \in \mathcal{N}_i} n_j$. We introduce projection matrices $W_i \in \{0,1\}^{n_{\mathcal{N}_i} \times n}$, such that $x_{\mathcal{N}_i} = W_i x$.

All the following variables are deviations from the nominal operation point. The generator dynamics $i \in \mathcal{G}$ can be approximated with the following linear model [4]

$$\begin{aligned}
\dot{P}_i^M &= -\frac{1}{T_i}(P_i^M - P_i^C), \\
\dot{\omega}_i &= -\frac{1}{M_i}\left(D_i \omega_i + P_i^L - P_i^M + \sum_{(i,j) \in \mathcal{E}} P_{ij}\right), \\
\dot{\delta}_i &= \omega_i, \\
P_{ij} &= B_{ij}(\delta_i - \delta_j) = B_{ij}\delta_{ij}, \quad (i,j) \in \mathcal{E},
\end{aligned}$$

with the mechanical power P_i^M, the frequency deviation ω_i, the phase shift δ_i, the power change command P_i^C, the power load P_i^L and the line power flow P_{ij}. With the local state $x_i = [P_i^M, \omega_i, \delta_i] \in \mathbb{R}^3$, the control input $u_i = P_i^C \in \mathbb{R}$ and external disturbance $d_i = P_i^L \in \mathbb{R}$, the system can be written in the following form,

$$\dot{x}_i = A_{c,\mathcal{N}_i} x_{\mathcal{N}_i} + B_{c,i} u_i + G_{c,i} d_i, \quad i \in \mathcal{G},$$

where $A_{c,\mathcal{N}_i}$, $B_{c,i}$ and $G_{c,i}$ are suitably defined matrices. The load dynamics $i \in \mathcal{L}$ are described by the power balance equation

$$\dot{\delta}_i = \omega_i = -\frac{1}{D_i}\left(P_i^L + \sum_{(i,j) \in \mathcal{E}} P_{ij}\right).$$

Correspondingly, the local state $x_i = \delta_i \in \mathbb{R}$ describes the load dynamics with a linear coupled model:

$$\dot{x}_i = A_{c,\mathcal{N}_i} x_{\mathcal{N}_i} + G_{c,i} d_i, \quad i \in \mathcal{L}.$$

The overall linear system is stable and described by

$$\dot{x} = A_c x + B_c u + G_c d.$$

B. Discrete time Network Model

In order to apply distributed MPC, we require a distributed discrete time model of the power network. Power networks have stiff dynamics, with time constants $\tau_i < 0.1\ ms$ for the loads $\mathcal{L}$. Thus, standard explicit discretization schemes are unsuitable for distributed power networks.

The simulation of power networks is typically accomplished with the implicit Trapezoidal method [9], which is used here for the predictions. Correspondingly, the system is described by the linear equality constraint

$$\underbrace{\left(I_n - \frac{h}{2}A_c\right)}_{F} x^+ = \underbrace{\left(I_n + \frac{h}{2}A_c\right)}_{A} x + \underbrace{hB_c}_{B} u + \underbrace{hG_c}_{G} d, \tag{1}$$

with the step size h. Due to the distributed structure of A_c, B_c, G_c the dynamic equality constraint (1) has the following distributed structure

$$F_{\mathcal{N}_i} x_{\mathcal{N}_i}^+ = A_{\mathcal{N}_i} x_{\mathcal{N}_i} + B_i u_i + G_i d_i.$$

C. Real time Economic Dispatch

The control specifications for the real time economic dispatch can be separated into three goals. The first goal is to ensure transient constraint satisfaction for the local power and the line power flow:

$$P_i^M \in [P_i^{M,\min}, P_i^{M,\max}], \quad i \in \mathcal{G}, \tag{2}$$

$$P_{ij} \in [P_{ij}^{\min}, P_{ij}^{\max}], \quad (i,j) \in \mathcal{E}. \tag{3}$$

The second goal is to improve the economic performance. The economic stage cost $l(x)$ is given by

$$l(x) = \sum_{i \in \mathcal{G}} l_i(x_i) = \sum_{i \in \mathcal{G}} \alpha_i(P_i^M) + \beta\omega_i^2.$$

The economic cost consists of local nonlinear cost functions $\alpha_i(P_i^M)$,[1] which reflects the cost of generating power. Furthermore, large frequency deviations are penalized with a quadratic cost on frequency deviations $\beta\omega_i^2$, with a parameter $\beta > 0$. To minimize the transient economic cost, the MPC minimizes the predicted stage cost $l(x(\cdot))$, instead of simply stabilizing the optimal steady-state.

The final control goal is to drive the system to a steady-state operation where the nominal frequency is recovered and the power balance is satisfied. This is achieved by adding the constraint

$$\delta_i \in [\delta_{\min}, \delta_{\max}], \tag{4}$$

which implicitly introduces a transient average constraint on the frequency deviation and thus recovers the nominal frequency. The state constraints (2), (3), (4) can be modeled with the following coupling polytopic constraint sets

$$\mathcal{X}_{\mathcal{N}_i} = \{x_{\mathcal{N}_i} | C_{\mathcal{N}_i} x_{\mathcal{N}_i} \leq c_i\}, \quad i \in \mathcal{N}, \quad c_i \in \mathbb{R}^{p_i}.$$

We also introduce the overall state constraint set $\mathcal{X}$:

$$\mathcal{X} = \{x | W_i x = x_{\mathcal{N}_i} \in \mathcal{X}_{\mathcal{N}_i},\ \forall i \in \mathcal{N}\}.$$

[1] The nonlinearity α_i is often approximated with a positive definite second order Taylor approximation around the nominal economic dispatch:

$$\alpha_i(P_i^M) \approx a_i \left(P_i^M\right)^2, \quad a_i > 0.$$

Note that we require satisfaction of the state constraints at all time instances, i.e. $x(t) \in \mathcal{X}, \forall t \geq 0$, not just at stationary operation.

III. Distributed Economic MPC

A. MPC Formulation

The distributed economic MPC (DEMPC) scheme, which is employed in this work, is characterized by solving the following distributed optimization problem at each time step

$$\begin{aligned} \min_{u(\cdot),\ x(\cdot)} & \sum_{k=0}^{N-1} \sum_{i \in \mathcal{G}} l_i(x_i(k)) \qquad (5) \\ \text{st. } & Fx(k+1) = Ax(k) + Bu(k) + Gd(k) \\ & x(0) = x_0 \\ & x(k) \in \mathcal{X}, \quad k \in \{0, \ldots, N-1\} \end{aligned}$$

with the current state x_0 and the predicted state and input trajectories $x(\cdot)$, $u(\cdot)$. The solution of the optimization problem are state and input trajectories $x^*(\cdot, x_0),\ u^*(\cdot, x_0)$ that satisfy the constraints and minimize the economic stage cost over the prediction horizon $N \in \mathbb{N}$. We denote the resulting MPC feedback by $\mu_N(x) = u^*(0, x)$.

B. Implementation Details

The closed-loop operation of the MPC can be summarized by the following algorithm:

Online MPC, execute at every time step t
1. Measure the state $x(t)$ and predict load $d(t : t + N)$.
2. Solve the MPC optimization problem (5) with $x_0 = x(t)$.
3. Apply the control input: $u(t) = \mu_N(x(t))$.

Assuming a quadratic economic cost, the optimization problem (5) is a quadratic program (QP),[1] the distributed online solution of which will be discussed in detail in Section IV. In practice, neither the full state $x(t)$, nor predictions for the load $d(t : t + N)$ are available. Thus, an augmented distributed Luenberger observer should be used to estimate $x(t)$ and $d(t)$ based on the available measurements such as $(\omega_i,\ P_{ij})$.[2] The load d can be assumed constant over the prediction horizon N and small deviations can be treated as a model mismatch. In case of large load changes, there usually exists some predictive knowledge that can be directly incorporated in the load trajectory $d(t : t + N)$.

C. Stability and Performance Guarantees

For the following analysis, we assume that the disturbance d stays constant (otherwise steady-state operation is not well defined). By adding small costs $\delta_i^2,\ u_i^2$ to the stage cost $l(x)$, we get a strictly convex quadratic stage cost. Thus, a strict dissipativity/strong duality condition involving the system dynamics and the stage cost is satisfied [11], which is one of the fundamental properties employed in EMPC. This condition implies that the system is economically optimally operated at the optimal equilibrium (x^e, u^e), with

$$\begin{aligned} (x^e, u^e) = \arg\min_{x,u} \ & l(x, u) \qquad (6) \\ \text{st. } x \in \mathcal{X}, \quad & Fx = Ax + Bu + Gd. \end{aligned}$$

Under controllability assumptions on the power network, the DEMPC can guarantee practical asymptotic stability of the closed loop system, if we use a large enough prediction horizon N. The corresponding theoretical results for EMPC without terminal constraints can be found in [12], [13], which also include transient economic performance guarantees.

IV. MPC with inexact minimization

Now we show how the optimization problem (5) can be solved in a distributed fashion, discuss some challenges concerning real time computation constraints and provide suitable modifications to the optimization problem to enable constraint satisfaction despite inexact minimization. Different theoretical results considering inexact dual minimization in MPC can be found in [14], [15], [10].

A. Distributed Optimization

To solve the optimization problem (5), we consider distributed optimization based on dual decomposition, which only requires local communication and is hence suitable for large scale systems. The alternating direction method of multipliers (ADMM) [16] is a popular method that can often achieve reasonable results with a few iterations.

A corresponding formulation of the distributed MPC optimization problem has been provided in [17] and is adopted here. We define a shared variable $z = \{x(\cdot), u(\cdot)\}$ and local variables $y_i = \{x^i_{\mathcal{N}_i}(\cdot), u_i(\cdot)\},\ i \in \mathcal{N}$. The local variables contain the local input trajectory $u_i(\cdot)$ and neighboring state trajectories $x^i_{\mathcal{N}_i}(\cdot)$, as optimized by subsystem i. To ensure that we recover the original solution, a consistency constraint $x^i_i(\cdot) = x^j_i(\cdot)$ is used, which can be written as

$$E_i z = y_i, \quad i \in \mathcal{N}. \qquad (7)$$

The MPC optimization problem (5) can be written as

$$\begin{aligned} \min_{y,z} \sum_{i \in \mathcal{N}} J_i(y_i) = & \sum_{i \in \mathcal{N}} \sum_{k=0}^{N-1} l_i(x^i_i(k), u_i(k)) \qquad (8) \\ \text{st. } y_i \in \mathcal{Y}_i(x_{i,0}), \quad & y_i = E_i z, \quad i \in \mathcal{N} \end{aligned}$$

with the local convex constraint set $\mathcal{Y}_i(x_i)$:

$$\begin{aligned} \mathcal{Y}_i(x_i) = \{ & y_i | x_i(0) = x_i,\ x^i_{\mathcal{N}_i}(k) \in \mathcal{X}_{\mathcal{N}_i},\ 0 \leq k \leq N-1, \\ & F_{\mathcal{N}_i} x^i_{\mathcal{N}_i}(k+1) = A_{\mathcal{N}_i} x^i_{\mathcal{N}_i}(k) + B_i u_i(k) + G_i d_i(k)\}. \quad (9) \end{aligned}$$

The augmented Lagrangian $\mathcal{L}_{i,\rho}$ is given by

$$\mathcal{L}_{i,\rho}(y_i, z, \lambda_i) = J_i(y_i) + \lambda_i^\top (y_i - E_i z) + \frac{\rho}{2} \|E_i z - y_i\|_2^2,$$

with a tuning parameter $\rho > 0$ and the local dual variable λ_i. The distributed ADMM iteration to solve (8) is summarized by the following algorithm:

[2] Distributed Luenberger observers can be computed with distributed semi-definite programming (SDP). In [10], the state x and a randomly changing load d have been estimated for a nonlinear power network using this method.

Alternating Direction Method of Multipliers (ADMM)
for $p = 0, \dots$ until convergence

$y_i^{p+1} = \arg \min_{y_i \in \mathcal{Y}_i(x_{i,0})} \mathcal{L}_{i,\rho}(y_i, z^p, \lambda_i^p)$

communicate y_i^{p+1} to neighbors $j \in \mathcal{N}_i$

$z_i^{p+1} = \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} E_{ji}^\top (y_{ji}^{p+1} + \frac{1}{\rho}\lambda_{ji})$

communicate z_i^{p+1} to neighbors: $j \in \mathcal{N}_i$

$\lambda_i^{p+1} = \lambda_i^k + \rho(y_i^{p+1} - E_i z^{p+1})$

This iteration asymptotically converges to the optimal solution of the original optimization problem [16].

B. Inexact Distributed Economic MPC

Due to real time computation requirements, in general it is not feasible to compute the exact optimal solution in each time step t. Therefore, we stop the ADMM optimization once the following stopping condition is satisfied

$$\|E_i z - y_i\|_\infty \leq \epsilon, \quad \epsilon > 0, \tag{10}$$

which can be efficiently checked during the computation. We denote the input sequence resulting from the inexact minimization with initial state x_0 by $u_\epsilon(\cdot, x_0)$. The state trajectory $x_\epsilon(\cdot, x_0)$ from the inexact minimization is given by

$$x_{i,\epsilon}(k, x_0) = x_i^i(k).$$

Due to the inexact solution (10), the consistency constraints (7) are not satisfied and the inexact trajectories satisfy the following perturbed dynamic equation:

$$\begin{aligned} F_{\mathcal{N}_i} x_{\mathcal{N}_i,\epsilon}(k+1, x_0) = & A_{\mathcal{N}_i} x_{\mathcal{N}_i,\epsilon}(k, x_0) + B_i u_{i,\epsilon}(k, x_0) \\ & + G_i d_i(k) + w_i(k), \end{aligned} \tag{11}$$

with the bounded error

$$w_i(k) \in \mathcal{W}_i := \{w_i | \|w_i\|_\infty \leq v_i\},$$
$$v_i := 2\epsilon \left(\sum_{j \in \overline{\mathcal{N}}_i} \|A_{ij}\|_\infty + \sum_{j \in \overline{\mathcal{N}}_i} \|F_{ij}\|_\infty \right).$$

Correspondingly we define the overall error as

$$w \in \mathcal{W} = \mathcal{W}_1 \times \cdots \times \mathcal{W}_{|\mathcal{N}|}.$$

Now we discuss the implications of the inexact solution on the resulting closed-loop system and propose counter measures with corresponding guarantees.

To ensure constraint satisfaction despite these inexact dynamics, we propose using robust MPC methods to tighten the constraint set.[3] To study feasibility with this inexact DEMPC, we define the consolidated trajectory [14], [15].

Definition 1. *The consolidated trajectory $\overline{x}_\epsilon(\cdot, x_0)$ is the state trajectory consistent with the optimized input trajectory $u_\epsilon(\cdot, x_0)$ and the dynamic constraint:*

$$\begin{aligned} F\overline{x}_\epsilon(k+1, x_0) &= A\overline{x}_\epsilon(k, x_0) + Bu_\epsilon(k) + Gd(k), \\ & \qquad k \in \{0, \dots, N-1\}, \\ \overline{x}_\epsilon(0, x_0) &= x_0. \end{aligned}$$

In general, the consolidated trajectory $\overline{x}_\epsilon(\cdot, x_0)$ differs from the inexact state trajectory $x_\epsilon(\cdot, x_0)$, due to the error w in (11) resulting from the inexact solution (10). To ensure constraint satisfaction of the consolidated trajectory $\overline{x}_\epsilon(\cdot, x_0)$ despite the inexact minimization, we use constraint tightening techniques, which are typically employed in robust tube-based MPC [19] with the k-step support function:

$$\begin{aligned} \sigma_{\mathcal{W}}(a, k) = & \sup_{w \in \mathcal{W}^k} a^\top y(k), \\ & \text{st. } y(0) = 0, \\ & \quad Fy(l+1) = Ay(l) + w(l), \quad l = 1, \dots, k-1. \end{aligned}$$

The evaluation of the k-step support function amounts to solving a distributed linear program (LP) offline. The local tightened constraints $\overline{\mathcal{X}}_{\mathcal{N}_i,k}$ for the prediction step k

$$\overline{\mathcal{X}}_{\mathcal{N}_i,k} = \{x_{\mathcal{N}_i} | C_{\mathcal{N}_i,j} x_{\mathcal{N}_i} \leq \overline{c}_{i,j,k}, j = 1, \dots p_i\},$$

are computed according to

$$\overline{c}_{i,j,k} = c_{i,j} - \sigma_{\mathcal{W}}((C_{\mathcal{N}_i,j} W_i)^\top, k).$$

With this we can state the tightened optimization problem

$$\begin{aligned} \min_{u,x} & \sum_{k=0}^{N-1} l(x(k), u(k)) \\ \text{st. } & Fx(k+1) = Ax(k) + Bu(k) + Gd(k) \\ & x(0) = x_0 \\ & x_{\mathcal{N}_i}(k) \in \overline{\mathcal{X}}_{\mathcal{N}_i,k}, \quad i \in \mathcal{N}, \ k \in \{0, \dots, N-1\} \end{aligned} \tag{12}$$

which can also be solved using the ADMM algorithm, with tightened sets $\overline{\mathcal{Y}}_i(x_i)$ defined analogous to (9). To study the closed-loop properties of the inexact DEMPC, we have to specify the consolidated cost and the inexact solution.

Definition 2. *The open loop cost of a K-step input sequence $u(\cdot)$ and the initial state x_0 is given by*

$$J_K(x_0, u(\cdot)) = \sum_{k=0}^{K-1} l(\overline{x}(k)),$$
$$F\overline{x}(k+1) = A\overline{x}(k) + Bu(k) + Gd(k), \quad \overline{x}(0) = x_0.$$

Definition 3. *An (ϵ, η) approximate solution to the optimization problem* (12) *satisfies the stopping condition* (10) *with ϵ and has a suboptimality smaller than η:*

$$\underbrace{J_N(x_0, u_\epsilon(\cdot))}_{=:\mathcal{V}_{N,\epsilon}(x_0)} \leq \underbrace{J_N(x_0, u^*(\cdot))}_{=\mathcal{V}_N^*(x_0)} + \eta.$$

Here, $u_\epsilon(\cdot)$ is the inexact input trajectory based on the inexact solution of optimization problem (12) *and $u^*(\cdot)$ is the optimal solution to* (5). *The corresponding inexact MPC feedback is denoted by $\mu_{N,\epsilon}(x) = u_\epsilon(0, x)$.*

[3] Since the power network is already stable, there is no need to use an additional stabilizing feedback. Methods to compute distributed stabilizing controllers can be found in [18], [10].

Bounds on the suboptimality η for $(\epsilon, \cdot)$ solutions can be established based on the constraint tightening and the dual variables [14], due to the feasibility with respect to the original optimization problem. The following result establishes properties for the closed-loop system with the inexact MPC based on the (ϵ, η) solution and nominal controllability assumptions on the system dynamics. These controllability assumptions imply that we can control the system locally at the optimal steady-state x^e and that it is possible to steer the system close to the optimal steady-state x^e in a finite number of steps without violating the constraints.

Theorem 4. *Assume that in each time step the inexact MPC feedback $\mu_{N,\epsilon}(x)$ is based on a (ϵ, η) solution to optimization problem* (12) *with $x_0 = x(t)$. Then the closed loop system*

$$Fx^+ = Ax + B\mu_{N,\epsilon}(x) + Gd \tag{13}$$

satisfies the constraints, i.e. $x(t) \in \mathcal{X},\ t \geq 0$. If the nominal conditions for stability (Theorem. 3.7 [12]) are satisfied with respect to the tightened constraints, the optimal steady-state x^e is practically asymptotically stable for the resulting closed-loop system (13)*.*

Proof. For reasons of brevity, the proof is only sketched. Details can be found in [10].
Feasibility: The feasibility of the consolidated trajectory $\overline{x}_\epsilon$ is a direct consequence of the constraint tightening and the ϵ accuracy ($w \in \mathcal{W}$). The closed-loop constraint satisfaction follows from the repeated feasibility of the consolidated trajectory, i.e.

$$x(t+1) = \overline{x}_\epsilon(1, x(t)) \in \mathcal{X}.$$

Stability: The stability analysis with suboptimality η is an extension of the nominal derivations in [13]. Abbreviate $f_d(x,u) = F^{-1}(Ax + Bu + Gd)$. From the dynamic programming principle and Definition 3 we know

$$\begin{aligned}\mathcal{V}_N^*(x) =& l(x, \mu_N(x)) + \mathcal{V}_{N-1}^*(f_d(x, \mu_N(x))\\ \leq& l(x, \mu_{N,\epsilon}(x)) + \mathcal{V}_{N-1}^*(f_d(x, \mu_{N,\epsilon}(x)))\\ \leq& l(x, \mu_{N,\epsilon}(x))\\ &+ J_{N-1}(f_d(x, \mu_{N,\epsilon}(x)), \{u_\epsilon(1), \ldots, u_\epsilon(N-1)\})\\ =& J_N(x, u_\epsilon(\cdot)) = \mathcal{V}_{N,\epsilon}(x) \leq \mathcal{V}_N^*(x) + \eta,\end{aligned}$$

which implies

$$l(x, \mu_{N,\epsilon}(x)) \leq \mathcal{V}_N^*(x) - \mathcal{V}_{N-1}^*(f_d(x, \mu_{N,\epsilon}(x))) + \eta. \tag{14}$$

Starting from (14), we can use the nominal derivations in [13] to show a decrease condition on a suitably defined Lyapunov function $\tilde{\mathcal{V}}_N(x)$:

$$\tilde{\mathcal{V}}_N(f_d(x, \mu_{N,\epsilon}(x))) - \tilde{\mathcal{V}}_N(x) \leq -\alpha_l(\|x\|) + \delta(N) + \eta,$$

which implies practical asymptotic stability. □

Remark 5. *Based on the decrease condition on $\mathcal{V}_N^*$ under the suboptimal MPC feedback $\mu_{N,\epsilon}$, transient economic performance guarantees can be derived [10], which is an extension of the nominal transient performance guarantees in Theorem 4.1 [12].*

V. Simulation Results

A. Simulation Setup

To demonstrate the effectiveness of the DEMPC approach, we compare it to the unified control (UC) in simulation scenarios with the IEEE 39 bus system, which can be seen in Figure 1. UC is a continuous time primal-dual algorithm, that asymptotically stabilizes the optimal steady-state (6). The control parameters for UC and the simulation model are taken from [4] with minor modifications[4].

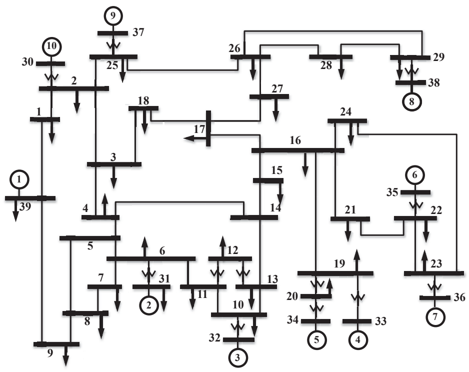

Fig. 1. IEEE 39 bus system: New England (taken from [4]).

The step size for the trapezoidal discretization is $h = 0.1\ s$ and the prediction horizon is $N = 20$, which is equivalent to a 2-second prediction horizon. The frequency convergence is enforced with $|\delta_i| \leq 40\ pu$. The stage cost is chosen as

$$l_i(x_i, u_i) = a_i \left(P_i^M\right)^2 + \beta\omega_i^2 + \gamma_i(\delta_i^2 + a_i u_i^2), \quad i \in \mathcal{G}$$

with a_i based on the economic dispatch, $\beta = 2 \cdot 10^{-2}$ and $\gamma_i = 10^{-4}$. The corresponding optimal steady-state is the same as in [4]. We use the accuracy $\epsilon = 10^{-3}$ for the stopping condition and the penalty $\rho = 0.1$. The following simulations were obtained using the original continuous time model, while the DEMPC controller design uses a discretized model. The constraints are tightened with an error set $\mathcal{W}$, that incorporates both model mismatch and inexact minimization.

B. Economic Performance

We first investigate the economic performance under a large load change, without considering constraints. We consider a sudden load change $P_{30}^L = 18\ pu$, compensation of which requires an increase in the overall power generation. The resulting trajectories can be seen in Figure 2. The UC works with a continuous controller, which leads to a slow but smooth power trajectory $P^M(\cdot)$. The DEMPC swiftly achieves frequency synchronization and converges to the optimal frequency. Once the constraint on δ becomes active, the nominal frequency is restored. The constraint on δ allows us to tune the nominal frequency restoration independently in contrast to UC. The DEMPC achieves a smaller economic cost and faster frequency synchronization.

[4]To ensure realistic simulation for the loads, a damping $D_{\mathcal{L}}$ equal to the smallest generator damping is added. To improve the numerical conditioning of the online optimization, $\delta_i,\ P_i^C$ are scaled by 10^2 and 10^{-3} respectively.

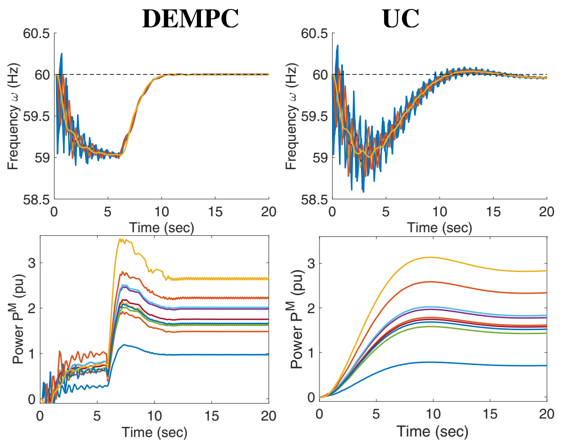

Fig. 2. Economic Performance

C. Constraint Satisfaction

Now we include hard constraints on the line flow P_{ij} and consider a similar scenario. We assume, that the prediction of the large load change is available 1s before the change. For illustration we consider a limit of 6 pu on the lines $(1,2)$, $(2,3)$, $(2,25)$, which distribute the load from bus 2 to the network. In Figure 3 we can see that the DEMPC reacts before the load change occurs due to the predictive nature and successfully keeps the line power flow within the prescribed bounds. The UC has strong oscillations in the line power flow and only satisfies the constraint asymptotically ($\sim$100s). The predictive capability of the MPC allows us to incorporate knowledge about future changes in the load, which is significantly more difficult to incorporate in other controllers.

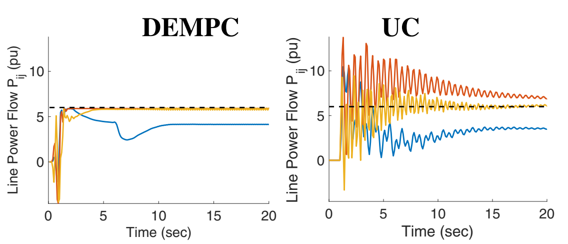

Fig. 3. Constraint Satisfaction

VI. CONCLUSIONS

We have shown that DEMPC can be used to solve the real time economic dispatch problem for power networks. The proposed DEMPC stabilizes the dynamics, restores the nominal frequency, achieves constraint satisfaction and guarantees transient economic performance under inexact minimization. In the simulations, we can see a potential improvement by considering the dynamic optimization method MPC instead of relying on primarily steady-state optimization methods like UC. The main limitation of using MPC for fast large scale systems, like power networks, is the real time optimization. We have addressed this problem with three methods: implicit prediction models, dual distributed optimization and robust MPC compensation for inexact minimization, thus allowing for a real time application of the proposed DEMPC algorithm.

DEMPC under inexact minimization still requires further research, especially moving from sufficient stopping conditions to prior bounds on the number of required iterations. To demonstrate the effectiveness of the DEMPC approach for power systems, a more comprehensive simulation test with nonlinear dynamics and random fluctuations would be useful.

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