Distributed access control of volatile renewable energy resources

Minghui Zhu, Na Li, Wenbo Shi and Rajit Gadh

Abstract— **In this paper, we develop a general framework of distributed access control which allows for anytime connections and disconnections of volatile renewable energy resources in the power grid. We formally prove that the hard state constraints are satisfied all the time and the asymptotic stability is reached if connections and disconnections only happen for a finite number of times. Our framework and analysis leverage the tools of input-to-state stability and small gain theorems in nonlinear systems. The performance of our framework is illustrated by a case study on frequency control with wind integration.**

I. Introduction

In the power grid, a fundamental objective is to reliably balance power generations and demands, simultaneously ensuring the transient performance and asymptotic stability of the underlying power grid. This objective has been facilitated by the wide deployment of advanced sensing, communication, computing and control infrastructures as well as the deep penetration of renewable energy. However, on the other hand, the large population of distributed renewable energy resources, including wind turbines, solar arrays, PHEV fleets and μ-CHPs, is also significantly challenging angle and voltage stability. Firstly, distributed energy resources are spatially distributed and thus distributed control becomes necessary. Secondly, distributed energy resources are managed by heterogeneous authorities, and the system operator may only have limited control on the deregulated power system. Thirdly, distributed energy resources might switch between the modes of grid-connected and island at will. The induced dynamic grid topologies potentially cause large overshoots of dynamic transient responses and further the violations of hard state constraints. Fourthly, wind and solar generation is not only intermittent but also highly volatile. The induced disturbances may be propagated and amplified through transmission lines, causing system-wide instability.

Literature review. The effects of external disturbances on dynamic performance of the power grid have been receiving increasing attention, including sensitivity analysis [5], [6], reachability analysis [1], [9] and set-theoretic method [3]. However, most of them do not consider controller synthesis and their approaches are centralized.

Distributed control of the power grid has been extensively studied. The classic distributed control includes power system stabilizer (PSS) and automatic generation control (AGC) [4], [10], [11], [14], [15]. The recent papers [7], [8] investigate AGC with wind integration. However, the papers lack of formal analysis, ignore hard state constraints and focus on distributed controller synthesis instead of access control in deregulated power systems.

Contributions. In this paper, we develop a general framework of distributed access control for interconnected power systems with renewable integration. In particular, our framework allows for anytime connections of distributed energy resources if distributed dynamic systems satisfy certain properties of input-to-state stability and their initial states are inside certain regions. On the other hand, our framework also allows for anytime disconnections of distributed dynamic systems when the associated disturbances are larger than a certain threshold. The decisions on connections and disconnections are based on simple rules which only require local information. We formally prove that the hard state constraints are satisfied all the time and the asymptotic stability is reached if connections and disconnections only happen for a finite number of times. Our framework and analysis leverage the tools of input-to-state stability and small gain theorems in nonlinear systems. The performance of our framework is illustrated by a case study on frequency control with wind integration. Due to the space limitation, the analysis is omitted in the current paper and provided in [16].

A. Notations

Denote the supremum norm of the truncation of $u(t)$ in $[t_1, t_2]$ by $\|u\|_{[t_1,t_2]} \triangleq \sup_{t_1 \leq t \leq t_2} \|u(t)\|$[1]. A continuous function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ function if $\gamma(0) = 0$ and it is strictly increasing. The function $\beta(s,t) : \mathbb{R}_{\geq 0} \times \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ belongs to class $\mathcal{KL}$ if for each fixed t, the function $\beta(\cdot, t)$ is a class $\mathcal{K}$ function, and for each fixed s, $\beta(s, \cdot)$ is deceasing and $\lim_{t\to+\infty} \beta(s,t) = 0$. A function $\gamma : [a,b] \to [c,d]$ is a contraction mapping if $\gamma(s) < s$ for all $s \in [a,b]$. $\mathbf{1}$ is the indicator function.

II. General framework

In this section, we will develop a framework to solve a class of distributed access control problem generalized from the one introduced in Section III. The general framework is able to deal with nonlinear dynamic systems subject to bounded exogenous disturbances.

[1]In this paper, we will use the infinite norm for the vectors and matrices.

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A. System model

The power system includes a set of buses $\mathcal{N} \triangleq \{1,\cdots,N\}$ and a set of transmission lines $\mathcal{E}$ connecting those buses. We denote $(i,j)\in\mathcal{E}$ if there is a line between i and j and $\mathcal{N}_i \triangleq \{j\in\mathcal{N} : (j,i)\in\mathcal{E}\}$. In this paper, we consider the case where a bus i can be connected or disconnected from the main interconnected system (main grid), which induces a dynamically changing topology. At any given time instant t, let $\mathcal{N}(t)\subset\mathcal{N}$ denote the buses that are connected within the main grid and $\mathcal{E}(t)\subset\mathcal{E}$ denote the transmission lines induced by $\mathcal{N}(t)$, i.e. $\mathcal{E}(t)\triangleq\{(i,j)\in\mathcal{E} : i,j\in\mathcal{N}(t)\}$. Denote $\mathcal{N}_i(t)\triangleq\{j\in\mathcal{N}(t)\mid (j,i)\in\mathcal{E}(t)\}$. Each bus is connected to a synchronous machine and the dynamics is governed by:

$$\dot{x}_i(t) = f_i\left(\{x_j(t)\}_{j\in\mathcal{N}_i(t)}, u_i(t), d_i(t), t\right), \qquad (1)$$

where $x_i(t)\in\mathbb{R}^{n_i}$ is the state, $u_i\in\mathbb{R}^{p_i}$ is the input, $d_i\in\mathbb{R}^{m_i}$ is the external disturbance. The quantity d_i stands for local time-varying uncertainty; e.g., $d_{L_i}-d_{ren_i}$ in system (7) in Section III.

The group of systems (1) is coupled via the system states, and these couplings are dynamically changing. We consider the scenario where a bus i is connected within the main grid at time t, i.e., $i\in\mathcal{N}(t)$, if only if $a_{c_i}(t)b_{c_i}(t)=1$ where $b_{c_i}(t)\in\{0,1\}$ is an exogenous signal and $a_{c_i}(t)\in\{0,1\}$ is an access control signal. The focus of this paper is to determine a local control protocol for $a_{c_i}(t)$ to guarantee the constraint enforcement and system stability of the main grid.

System (1) is a generalization of system (7) in Section III. We assume that there is already a controller u_i under which dynamic system (1) is relatively stable with respect to its neighboring states and disturbance. This is formalized by the notion of input-to-state stability (ISS, for short) as follows:

Assumption 2.1 (Input-to-state stability): There is a controller u_i such that dynamic system (1) is ISS with respect to x_j and d_i; i.e., there exist class $\mathcal{KL}$ function $\beta_i(\cdot,\cdot)$ and class $\mathcal{K}$ functions $\gamma_{ij}(\cdot)$ and $\gamma_{id}(\cdot)$ such that for all $x_i(t_0)\in X_{ii}$ the following holds for all $t\geq t_0$:

$$\begin{aligned}\|x_i(t)\| \leq &\max\{\beta_i(\|x_i(t_0)\|, t-t_0), \gamma_{id}(\|d_i\|_{[t_0,t]}),\\ &\max_{j\in\mathcal{N}_i(t)}\{\gamma_{ij}(\|x_j\|_{[t_0,t]})\}\},\end{aligned} \qquad (2)$$

where the functions of β_i, γ_{ij} and γ_{id} are time independent.

In (2), the term of β_i characterizes the transient performance of system (1) and the decreasing rate of the trajectory. The remaining terms capture the ultimate bound on $\|x_i(t)\|$. The notion of ISS was first proposed in the seminal paper [13]. Since then, ISS has become a powerful tool to analyze the stability of nonlinear systems.

B. Problem statement: distributed access control

We assign agent $\mathbf{M}_i$ to monitor dynamic system i. Distributed access control problem is to design a rule for each monitor agent $\mathbf{M}_i$ which decides on $a_{c_i}(t)$ by only using local information including the states of $x_i(t)$ and $x_j(t)$ with $j\in\mathcal{N}_i(t)$ such that $x_i(t)\in X_{ii}$ for all $t\geq t_0$ and the states in $\mathcal{N}(t)$ is stable.

C. Preliminary steps

Before presenting our distributed access controller, we will first introduce a set of auxiliary notations. Let $\Delta_{ij}\in\mathbb{R}_{>0}$ such that $\{x_i\in\mathbb{R}^{n_i}\mid \|x_i\|\leq\gamma_{ij}(\Delta_{ij})\}\subseteq X_{ii}$, $\delta_i\in\mathbb{R}_{>0}$ such that $\{x_i\in\mathbb{R}^{n_i}\mid\|x_i\|\leq\gamma_{id}(\delta_i)\}\subseteq X_{ii}$. We further define the set $\hat{X}_i$ and the scalars $\hat{\delta}_i$ as follows:

$$\hat{X}_i \triangleq \{x_i\in X_{ii}\mid \beta_i(\|x_i\|,0) < \min_{j\in\mathcal{N}_i}\Delta_{ji}\},$$

$\hat{\delta}_i<\delta_i$ is such that $\forall s\in[0\ \ \hat{\delta}_i)$, it holds that

$$\max_{j\in\mathcal{N}_i}\gamma_{ij}\circ\gamma_{jd}(s) < \min_{j\in\mathcal{N}_i}\Delta_{ji}, \quad \forall t\geq t_0.$$

The above notations are defined for the purposes of the introduction of our distributed access control algorithm and the analysis in Theorem 2.1.

D. Distributed access controller

Roughly speaking, the constraint enforcement and system stability are guaranteed if the following sufficient conditions hold. Each gain function γ_{ij} is a contraction mapping as well as the initial states and disturbances are inside the sets captured by $\hat{X}_i$ and $\hat{\delta}_i$. The sufficient conditions can be verified in a completely distributed manner. In addition, the verification of the sufficient conditions is rather simple and this feature enables real-time access control. The controller is formally stated in Algorithm 1. Here, we assume that $\mathbf{M}_i$ knows X_{ii}, β_i, γ_{ij} and γ_{id}.

Algorithm 1 Distributed access controller

Require: Each monitor agent $\mathbf{M}_i$ computes $\hat{X}_i$, $\hat{\delta}_i$ by using X_{ii}, β_i, γ_{ij} and γ_{id}. Initially, $x_i(t_0)\in\hat{X}_i$ for all $i\in V$.

Ensure: At each $t\geq t_0$, each monitor agent $\mathbf{M}_i$ executes the following steps:

1: Sets $a_{c_i}(t)=0$ if the following both hold: (i) $b_{c_i}(t)=1$; (ii) either $x_i(t)\notin\hat{X}_i$ or the following holds:

$$\begin{aligned}\|x_i(t)\| = &\max\{\beta_i(\|x_i(t_0)\|, t-t_0), \gamma_{id}(\hat{\delta}_i),\\ &\max_{j\in\mathcal{N}_i(t)}\{\gamma_{ij}(\|x_j\|_{[t_0,t]})\}\}.\end{aligned} \qquad (3)$$

2: Sets $a_{c_i}(t)=1$ if $b_{c_i}(t)=1$ and all of the following are satisfied:
   1) dynamic system i satisfies Assumption 2.1;
   2) $x_i(t)\in\hat{X}_i$;
   3) γ_{ij} is a contraction mapping for each $(i,j)\in\mathcal{E}$ and $j\in\mathcal{N}(t)$;
   4) γ_{ji} is a contraction mappings for each $(j,i)\in\mathcal{E}$ and $j\in\mathcal{N}(t)$.

Denote by Ξ the set of time instants when there is a change of $\mathcal{N}(t)$, i.e. $\Xi\triangleq\{t\geq t_0 : \lim_{\tau\to t^-}\mathcal{N}(\tau)\neq\mathcal{N}(t)\}$. The performance of the distributed access controller is stated as follows.

Theorem 2.1: The distributed access controller ensures that $x_i(t)\in\hat{X}_i$ for all $i\in\mathcal{N}(t)$ and $t\geq t_0$ (even when $|\Xi|$

is infinite). If $|\Xi|$ is finite, then there is class $\mathcal{KL}$ function $\tilde{\beta}$ and class $\mathcal{K}$ function $\tilde{\gamma}$ such that for all $t \geq \xi$, it holds that:

$$\|\tilde{x}(t)\| \leq \max\{\tilde{\beta}(\|\tilde{x}(\xi)\|, t-\xi), \tilde{\gamma}(\sigma)\},$$

where $\tilde{x}(t) \triangleq \{x_i(t)\}_{i\in\mathcal{N}(\xi)}$, $\sigma \triangleq \max_{i\in V}\{\hat{\delta}_i\}$ and ξ is the largest time instant in Ξ.

Proof: The proof is shown in Technique report **[?]** ■

III. Numerical simulation: frequency control with wind integration

In this section, we will apply our general framework in Section II to a frequency control problem. The performance of the distributed access controller will be illustrated by numerical simulations. The parameters used in this section are summarized in Tables I, II and III.

A. Power system model

We consider a power system where each bus denotes a control authority, which may consist of a variety of generators and/or loads. It is a common practice to lump all the generators (resp. loads) of a control authority as a single generator (resp. load). At each control authority, we assume that there is a mechanical generation P_{M_i}, wind generation P_{ren_i} and load P_{L_i}. The control authorities are connected with each other through the power flow P_{ij}. The scheme is shown in Figure 1

1) Linearized dynamics: Similar to [11], we will adopt a linearized model of synchronous generators for each control authority. The notation Δ is used to indicate a deviation from (potentially time-varying) nominal value $[w^*, \theta_i^*, P_{L_i}^*, P_{M_i}^*, P_{v_i}^*, P_{ren_i}^*, P_{ref_i}^*]^T$. For example, Δw_i represents the deviation of angular frequency from the constant set-point w^*; e.g., 60 Hz.

At time instant t, the state-space model of control authority i linearized at the set-point $[w^*, \theta_i^*, P_{L_i}^*, P_{M_i}^*, P_{v_i}^*, P_{ren_i}^*, P_{ref_i}^*]^T$ is given by the following:

$$\begin{aligned}\frac{d\Delta w_i}{dt} &= -\frac{1}{M_i}\big(D_i\Delta w_i + \sum_{j\in\mathcal{N}_i(t)}\Delta P_{ij} \\ &\quad - \Delta P_{M_i} + \Delta P_{L_i} - \Delta P_{ren_i}\big),\\ \frac{d\Delta\theta_i}{dt} &= 2\pi\Delta w_i,\\ \frac{d\Delta P_{M_i}}{dt} &= -\frac{1}{T_{CH_i}}\big(\Delta P_{M_i} - \Delta P_{v_i}\big),\\ \frac{d\Delta P_{v_i}}{dt} &= -\frac{1}{T_{G_i}}\big(\Delta P_{v_i} + \frac{1}{R_i}\Delta w_i - \Delta P_{ref_i}\big), \end{aligned} \quad (4)$$

and the power flow between control authorities i and j is modeled by:

$$\Delta P_{ij}(t) = T_{ij}(\Delta\theta_i(t) - \Delta\theta_j(t)). \quad (5)$$

The first equation (4) is the swing dynamics and captures the frequency evolution. The second equation in (4) together with (5) describes the branch flow dynamics. The third and fourth equations in (4) stand for the dynamic system of turbine-governor which is controlled via the control command signal ΔP_{ref_i}.

Remark 3.1: The readers are referred to [11] for the detailed derivation of (4) and (5). As pointed out in [8], fast and low-magnitude wind disturbances can be considered as small disturbances for today's power grid. The fidelity of linearized dynamics subject to wind disturbances is numerically verified in [3]. •

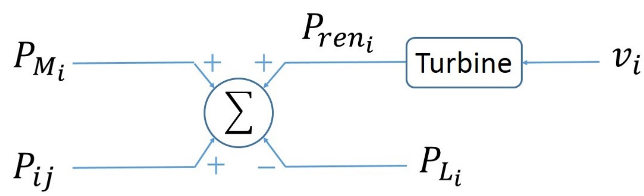

Fig. 1: Generation and loads of bus i.

2) Wind integration: The power injection from a wind turbine of control authority i is given by the following relations:

$$P_{w_i}(t) = \frac{1}{2}\rho\pi r_i^2 v_i(t)^3 \quad P_{ren_i}(t) = C_p P_{w_i}(t), \quad (6)$$

where $v_i(t)$ is wind speed and the quantity C_p is known as the turbine's power coefficient. The theoretic limit of C_p is 0.59 according to Betz's law, and it typically ranges from 0.2 to 0.4 (see [2], [12]). In the remainder of this paper, we will use $C_p = 0.59$.

Due to the fluctuation and uncertainty, $v_i(t)$ (resp. $P_{ren_i}(t)$) is probably different from the nominal value v_i^* (resp. $P_{ren_i}^*$). The discrepancy is modeled as a deterministic disturbance which is denoted by $d_{ren_i}(t) \triangleq \Delta P_{ren_i}(t) \triangleq P_{ren_i}(t) - P_{ren_i}^*$. Similarly, $d_{L_i}(t) = \Delta P_{L_i}(t) \triangleq P_{L_i}(t) - P_{L_i}^*$ is the disturbance induced by load deviation. Let $d_i(t) \triangleq d_{L_i}(t) - d_{ren_i}(t)$.

3) Compact model: For each control authority i, we denote its state by $x_i \triangleq [\Delta w_i \quad \Delta\theta_i \quad \Delta P_{M_i} \quad \Delta P_{v_i}]^T$ and control by $u_i \triangleq \Delta P_{ref_i}$. With this set of notations, we compactly write the model of (4) and (5) for control authority i as follows:

$$\dot{x}_i(t) = A_{ii}x_i(t) + \sum_{j\in\mathcal{N}_i(t)} A_{ij}x_j(t) + B_i u_i(t) + C_i d_i(t), \quad (7)$$

where the system matrices are given by:

$$A_{ii} \triangleq \begin{bmatrix} -\frac{D_i}{M_i} & -\sum_{j\in\mathcal{N}_i(t)}\frac{T_{ij}}{M_i} & \frac{1}{M_i} & 0 \\ 2\pi & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_{CH_i}} & \frac{1}{T_{CH_i}} \\ -\frac{1}{T_{G_i}R_i} & 0 & 0 & -\frac{1}{T_{G_i}} \end{bmatrix},$$

$$A_{ij} \triangleq \begin{bmatrix} 0 & \frac{T_{ij}}{M_i} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$B_i \triangleq \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_{G_i}} \end{bmatrix}, \quad C_i \triangleq \begin{bmatrix} -\frac{1}{M_i} \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The state $x_i(t)$ in (7) is subject to the constraint X_{ii} which must be enforced all the time.

TABLE I: Generator variables

w	angular frequency
θ	phase angle
P_M	mechanical power
P_M^*	set-point of mechanical power
P_L	actual load
P_L^*	forecasted load
P_v	stream valve position
P_v^*	set-point of stream valve position
P_{ij}	tie-line power flow between control authorities i and j
P_{ref}	reference power
P_{ref}^*	set-point of reference power
v	wind speed
P_w	wind power
P_{ren}	actual turbine power
P_{ren}^*	forecasted turbine power

TABLE II: Generator parameters adopted from Page 598 in [11]

M	angular momentum	10.0	s
D	load-damping constant	1.0	MW/Hz
R	speed droop	0.05	Hz/MW
T_{CH}	charging time constant	0.3	s
T_G	governor time constant	0.2	s
T_{ij}	tie-line stiffness coefficient	1.5	MW/rad

TABLE III: Variables and parameters of wind power

ρ	air density	1.225	kg/m^3
r_i	blade length	10.0	m
C_p	turbine's power coefficient	0.59	NA
v	wind speed	NA	m/s

B. Access control problem

Each control authority is associated with two binary signals: the requesting signal $b_{c_i}(t) \in \{0, 1\}$ and the access signal $a_{c_i}(t) \in \{0, 1\}$. In particular, if control authority i wants to connect to the power grid, then he submits a request; i.e., $b_{c_i}(t) = 1$. If the request is accepted by the system operator, then $a_{c_i}(t)$ is set to 1 and the control authority is connected with the main grid. In the meantime, some active control authorities passively become inactive from the main grid, $a_{c_i}(t) = 0$; e.g., the system operator has to disconnect faulty power generators or wind turbines. This induces time-varying network topologies.

The connections of new control authorities could potentially cause large overshoots of transient response and further violate the state constraints X_{ii}. In addition, the connections of new control authorities could potentially compromise the stability of the power grid due to oscillatory power exchanges. Furthermore, the transient performance and stability cannot be ensured if the system operator does not promptly respond to faulty power generators.

On the other hand, the disturbances induced by the volatility of wind power may be propagated and amplified through transmission lines, challenging the performance of the power grid. Hence, wind turbines should be disconnected when their volatilities are too large.

Overall, given the set of u_i of the control authorities, the access control problem for the system operator is to design a rule which decides the signals of $a_{c_i}(t)$ such that the constraint enforcement and asymptotic stability of system (7) can be guaranteed.

C. ISS stabilizability

Let us first identify u_i to satisfy Assumption 2.1. Consider dynamic system (7). One can verify that (A_{ii}, B_i) is controllable; i.e., one can choose K_i to arbitrarily place the poles of $\bar{A}_i \triangleq A_{ii} + B_i K_i$.

We now proceed to identify K_i such that control-free system (7) satisfies Assumption 2.1. Let us consider

$$\dot{x}_i(t) = \bar{A}_i x_i(t) + z_i(t), \tag{8}$$

where $z_i(t) \triangleq \sum_{j\in\mathcal{N}_i(t)} A_{ij}x_j(t) + C_i d_i(t)$. For (8), we have

$$\begin{aligned} \frac{d\|x_i(t)\|^2}{2dt} &= x_i(t)^T \dot{x}_i(t) = x_i(t)^T \bar{A}_i x_i(t) + x_i(t)^T z_i(t) \\ &\leq \lambda_{\max}(\bar{A}_i)\|x_i(t)\|^2 + \|x_i(t)\|\|z_i(t)\|. \end{aligned} \tag{9}$$

So, we have

$$\begin{aligned} \|x_i(t)\| \leq \max\{&2e^{\lambda_{\max}(\bar{A}_i)(t-t_0)}\|x_i(t_0)\|, \\ &\frac{(|\mathcal{N}_i(t)|+1)\max_{j\in\mathcal{N}_i(t)}\|A_{ij}\|}{|\lambda_{\max}(\bar{A}_i)|}\max_{j\in\mathcal{N}_i(t)}\|x_j\|_{[t_0,t]}, \\ &\frac{(|\mathcal{N}_i(t)|+1)\|C_i\|}{|\lambda_{\max}(\bar{A}_i)|}\|d_i\|_{[t_0,t]}. \end{aligned}$$

Let $\mathcal{N}_{i,\max} \triangleq \max_{i\in V}\sup_{t\geq t_0}|\mathcal{N}_i(t)|$ and the quantity $\mathcal{N}_{i,\max}$ is known *a priori*. A trivial upper bound on $\mathcal{N}_{i,\max}$ is $N-1$. Then the gain matrix $K_i(t)$ is chosen such that

$$|\lambda_{\max}(\bar{A}_i)| > (\mathcal{N}_{i,\max}+1)\max\{\max_{j\in\mathcal{N}_i(t)}\|A_{ij}\|, \|C_i\|\}.$$

As a result, control authority i is ISS with all the gain functions being linear and contractive.

D. Numerical simulations

In order to verify the effects of wind power, we assume that $d_{L_i}(t) = 0$ and $b_{c_i}(t) = 1$ for all $t \geq t_0$ and $i \in \mathcal{N}$ in the simulation. Consider $N = 5$ and the state constraint $X_{ii} = \{x_i \in \mathbb{R}^4 \mid \|x_i\| \leq 1\}$. The topology of the power system is shown in Figure 2.

Notice that $\mathcal{N}_{i,\max} = 4$. Choose the spectrum $\sigma(\bar{A}_i) = \{-3, -3, -3, -3\}$. Then, we have $\beta(s,t) = 2e^{-3t}s$ and $\gamma_{ij}(s) = \frac{1}{4}s$, $\gamma_{id}(s) = \frac{1}{6}s$. It further renders that $\Delta_{ij} = 4$, $\delta_i = \hat{\delta}_i = 6$ and $\hat{X}_i = \{x_i \in \mathbb{R}^4 \mid \|x_i\| \leq 1\}$.

The desired pole positions of $\sigma(\bar{A}_i) = \{-3, -3, -3, -3\}$ are given by $(s+3)^4 = s^4 + 12s^3 + 54s^2 + 108s + 81$.

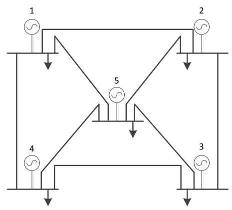

Fig. 2: Diagram of the power system topology

	State 1	State 2
State 1	0.9	0.1
State 2	0.5	0.5

TABLE IV: Transition matrix

Let $\gamma_c(A_{ii}) = A_{ii}^4 + 12A_{ii}^3 + 54A_{ii}^2 + 108A_{ii} + 81$. By the Ackermann's formula, we have $u_i(t) = K_i x_i(t)$ with

$$K_i = [0\ \ 0\ \ 0\ \ 1]Q_i^{-1}\gamma_c(A_{ii}),$$

where the matrix Q_i is given by: $Q_i \triangleq [B_i\ \ A_{ii}B_i\ \ A_{ii}^2B_i\ \ A_{ii}^3B_i]$.

The wind disturbances in the simulation are generated using a Markov model which has two states. In both states, the disturbance is a Gaussian random variable with a mean of zero. The first state has a standard deviation of 0.1 while the second state has a standard deviation of 0.5 to simulate the bursty large error. The transition matrix of the Markov model is shown in Table IV. The initial state is state 1. Figure 3 shows one realization of the disturbance at bus 1.

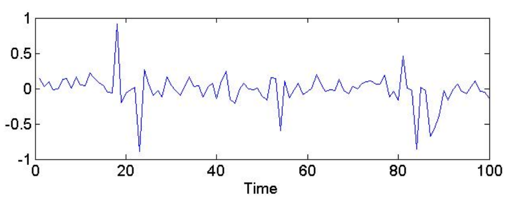

Fig. 3: Disturbance $d_{ren_1}(t)$ at bus 1.

Figure 4 shows the trajectory of $\|x(t)\|$ where the distributed access controller is not applied; i.e., $a_{c_i}(t) = 1$ for all $t \geq t_0$ and $i \in \mathcal{N}$. Figure 5 shows the trajectory of $\|\tilde{x}(t)\|$ where the distributed access controller is performed. In Figure 4, the hard state constraints are violated for most of the time and the overshoots at some time instants are very large. In Figure 5, the hard state constraints are always enforced. In Figure 5, $d_{ren_i}(t) = 0$ for all $t \geq 100$ and $i \in \mathcal{N}$. Then the trajectory exponentially decrease to zero after 100.

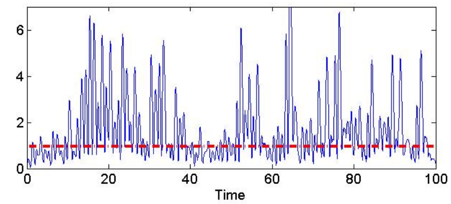

Fig. 4: The trajectory (blue) of $\|\tilde{x}(t)\|$ when the distributed access control is not applied. The horizontal red line at value 1 is plotted for the comparison with Figure 5.

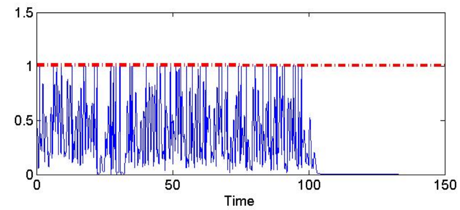

Fig. 5: The trajectory (blue) of $\|\tilde{x}(t)\|$ when the distributed access control is applied. The horizontal red line at value 1 is plotted for the comparison with Figure 4.

IV. Conclusion

We have proposed a distributed access controller to ensure the constraint enforcement and stability of the power grid with renewable integration. One of the future directions includes the investigation of random disturbances.

References

[1] M. Althoff, M. Cvetkovic, and M. Illic. Transient stability analysis by reachable set computation. In *IEEE PES International Conference and Exhibition on Innovative Smart Grid Technologies*, pages 1 – 8, 2012.

[2] A. Bertz. *Introduction to the theory of flow machines*. Oxford: Permagon press, 1966.

[3] Y.C. Chen and A.D. Dominguez-Garcia. A method to study the effect of renewable resource variability on power system dynamics. *IEEE Transactions on Power Systems*, 27(4):1978 – 1989, 2012.

[4] D.J. Glover, M.S. Sarma, and T.J. Overbye. *Power system analysis and design*. Cengage Learning, 2012.

[5] I.A. Hiskens and J. Alseddiqui. Sensitivity, approximation and uncertainty in power system dynamic simulation. *IEEE Transactions on Power Systems*, 21(4):1808 – 1820, 2006.

[6] I.A. Hiskens and M.A. Pai. Trajectory sensitivity analysis of hybrid systems. *IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications*, 47(2):204 – 220, 2000.

[7] M. Ilic and Q. Liu. *Toward Sensing, Communications and Control Architectures for Frequency Regulation in Systems with Highly Variable Resources*, pages 3–33. Control and Optimization Methods for Electric Smart Grids. Springer, 2011.

[8] M. Illic, M. Cvetkovic, K. Bachovchin, and Q. Liu. Modeling, analysis and control design complexities in future electrical energy systems. In *15 th International Power Electronics and Motion Control Conference*, pages LS2b.3–1 – LS2b.3–6, 2012.

[9] L. Jin, R. Kumar, and N. Elia. Reachability analysis based transient stability design in power systems. *International Journal of Electrical Power and Energy Systems*, 32(7):782 – 787, 2010.

[10] I. Kumar and D. Kothari. Recent philosophies of automatic generation control strategies in power systems. *IEEE Transactions on Power Systems*, 20(1):346 – 357, 2005.

[11] P. Kundur, N.J. Balu, and M.G. Lauby. *Power system stability and control*. McGraw-Hill, 1994.

[12] J.F. Manwell, J.G. McGowan, and A.L. Rogers. *Wind energy explained: theory, design and application*. John Wiley and Sons Ltd, 2002.

[13] E.D. Sontag. Smooth stabilization implies coprime factorization. *IEEE Transactions on Automatical Control*, 34(4):435 – 443, 1989.

[14] A.J. Wood and B.F Wollenberg. *Power Generation, Operation and Control*. J. Wiley and Sons, New York,, 1996.

[15] F.F. Wu, K. Moslehi, and A. Bose. Power system control centers: Past, present and future. *Proceedings of the IEEE*, 93(11):1890 – 1908, 2005.

[16] M. Zhu, N. Li, W. Shi, and R. Gadh. Distributed access control volatile renewable energy resources. Technical report, Pennsylvania State University, 2013.