Dynamic Detection of Transmission Line Outages Using Hidden Markov Models

Qingqing Huang, Leilai Shao, and Na Li

***Abstract*—We study the problem of detecting transmission line outages in power grids. We model the time series of power network measurements as a hidden Markov process, and formulate the line outage detection problem as an inference problem. Due to the physical nature of the line failure dynamics, the transition probabilities of the hidden Markov Model are sparse. Taking advantage of this fact, we further propose an approximate inference algorithm using particle filtering, which takes in the times series of power network measurements and produces a probabilistic estimation of the status of the transmission line status. We then assess the performance of the proposed algorithm with case studies. We show that it outperforms the conventional static line outage detection algorithms and is robust to both measurement noise and model parameter errors.**

***Index Terms*—Cascading failures, fault diagnosis, inference, transmission networks.**

I. Introduction

FAULT detection and diagnosis play important roles in control and protection for many systems. In this paper, we study the problem of detecting transmission-line outages in power networks. Transmission lines form a vital part of the power grid, as they provide the means to transfer electric power from power plants to end users. In order to ensure reliable operations, the transmission lines are densely interconnected in the transmission network. Due to the rapidly growing economics and technologies, the increasing electric demand is pushing the grid to operate the transmission lines close to their operating limits, making the entire transmission network vulnerable to disturbances. Unexpected events, such as sudden changes in power generations or loads, a breaker failure, a tree fall, or a lightning strike, can make the transmission lines inoperative. Outage, namely a transmission line being disconnected from the grid, is one of the most common faults. Moreover, the outage of a single transmission line, if not detected and treated quickly, may cascade into the breakdown of multiple lines in a few minutes and eventually lead to a costly grid-wide outage in less than an hour [1].

Manuscript received October 28, 2014; revised February 15, 2015; accepted June 29, 2015. Date of publication August 04, 2015; date of current version April 15, 2016. Paper no. TPWRS-01494-2014.

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Digital Object Identifier 10.1109/TPWRS.2015.2456852

Transmission protection systems are designed to identify the locations of faults and to isolate the faulted parts from the rest of the grid. In power systems, the goal is to keep the power system stable by isolating only the faulted sections while leaving as much of the network as possible in operation. Therefore, one of the key requirements for the protection system is to detect the faults promptly and accurately. However, due to the large scale of power grids, the highly nonlinear system dynamics, and the limited and noisy power system measurements, fast detection of transmission line outages remains a challenge.

There have been growing research efforts investigating different aspects of fault diagnosis in power systems based on static and dynamic state estimation. However, to the best of the authors' knowledge, the problem of detecting which transmission lines are disconnected from the grid is typically formulated as *static* hypothesis testing/optimization problems. Here, static refers to the "quasi-static" assumption that the interconnected grid has reached a stable state after all outage events and remains unchanged thereafter. Under this assumption, given the readings from phasor measurement units (PMUs), the network topology that offers the minimum fitting error is the maximum-likelihood estimator for the transmission line status. In order to bypass the combinatorial complexity, in [2]–[4], the authors applied compressed sensing inspired techniques to convexify the problem. In [5], the authors focused on detecting double line outages and discussed the problem of indistinguishable outages due to incomplete PMU measurements. In [6], the authors adopted the quickest detection framework to study this problem. However, these approaches overlook the underlying causal dynamics of cascading failure in the transmission networks, whereas, in this paper, we incorporate the information of temporal and spatial dependence between line outages. The proposed algorithm aims to improve the accuracy of line outage detection as well as to recognize the line status states that are indistinguishable based on static measurements.

In this paper, we focus on identifying the location and timing of the line outages using sequences of power system measurements, for example readings from supervisory control and data acquisition (SCADA) system and/or PMU readings of voltages and currents. We model the evolution of the transmission-line status as an unobservable Markov chain. The power system measurements, which reflect the line status, are thus modeled as the output process of a hidden Markov model (HMM). We formulate the problem of line outage detection as an HMM inference problem, i.e., estimating the hidden state of line status based on the observed output process of the HMM. Due to the physical nature of the line failure dynamics, the transition probabilities are sparse. Taking advantage of this fact, we propose an approximate inference algorithm using particle filtering, which has the following favorable properties.

1) It is resilient to noisy PMU readings which may be due to time skews, communication failure, parameter uncertainty, and infrequent instrument calibration. The proposed algorithm is able to aggregate information over time and, thus, is more robust to the measurement noise.
2) We incorporate our knowledge about the line failure dynamics into the model. This information allows us to rule out the line status configurations that are inconsistent with the physical laws which govern the cascading failure dynamics, and helps to distinguish the configurations which offer the same instantaneous readings.
3) The algorithm can better backtrack the initial cause of the failures as well as the cascading failure path over time.
4) The computational complexity is only linear in the number of particles. The particle filter based approximate algorithm exploits the sparsity of transition probabilities so that only a small number of particles are needed. This is meaningful for real systems with a large number of lines and exponentially many line status configurations.

In this paper, instead of modeling the transient dynamics of the power systems, we use a quasi-dynamical model to describe the temporal and spatial dependence between line outages, which is based on the static ac power flow and a probabilistic line failure model. However, we expect that the general ideas can be applied to more complicated and realistic models, and the above properties are likely to remain.

II. Preliminaries

A. AC Power Flow Model

We consider a transmission network consisting of N buses denoted by $\mathcal{N} = \{1, 2, \dots, N\}$ and E transmission lines denoted by $\mathcal{E} = \{1, 2, \dots, E\}$. We use $n \in \mathcal{N}$ to index the bus n and $e \in \mathcal{E}$ to index the transmission line e. We also refer a line e as mn where $m \in \mathcal{N}$ and $n \in \mathcal{N}$ are the two buses it connects. For each bus m, let $\mathcal{N}_m$ denote all of the buses that it is connected to. As a default, we assume $m \notin \mathcal{N}_m$.

First, we derive the bus admittance matrix from the equivalent π model [2] as shown in Fig. 1. For each line $mn \in \mathcal{E}$, denote the series admittance by y_{mn} and the total charging susceptance by $b_{c,mn}$. For each bus m, denote the shunt susceptance by $b_{s,mm}$ and let $y_{mm} := j(b_{s,mm} + \sum_{n \in \mathcal{N}_m} b_{c,mn}/2)$. We define the bus admittance matrix $Y = G + jB$ such that the diagonal entries $Y_{mm} = \sum_{n' \in \mathcal{N}_m} y_{mn'} + y_{mm}$ and the off-diagonal entries $Y_{mn} = -y_{mn}$ for $n \in \mathcal{N}_m$. Then, the polar presentation of the power flow equations yields is given by

$$P_m = \sum_{n \in \mathcal{N}} V_m V_n (G_{mn} \cos\theta_{mn} + B_{mn} \sin\theta_{mn}) \quad (1)$$

$$Q_m = \sum_{n \in \mathcal{N}} V_m V_n (G_{mn} \sin\theta_{mn} - B_{mn} \cos\theta_{mn}) \quad (2)$$

where V_m denotes the voltage magnitude at bus m, $\theta_{mn} := \theta_m - \theta_n$ denotes the phase difference on transmission line mn, and $\mathcal{S}_m := P_m + jQ_m$ denotes the power injection at bus m.

Given the system power injection profile $\mathcal{S}$ and system admittance matrix Y, the complex voltages $\mathcal{V}_m := V_m e^{j\theta}$ can be solved through (1)–(2), as a standard ac power flow problem [7]. Invoking Ohm's and Kirchoff's laws, the complex current $\mathcal{I}_{mn}$ and power flow on transmission line mn are given by

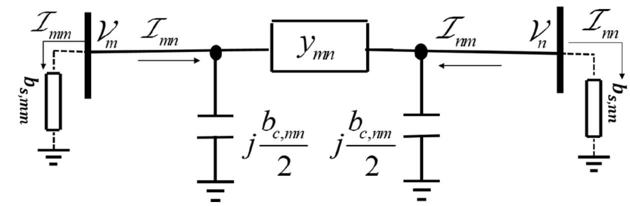

Fig. 1. Equivalent π model for a transmission line.

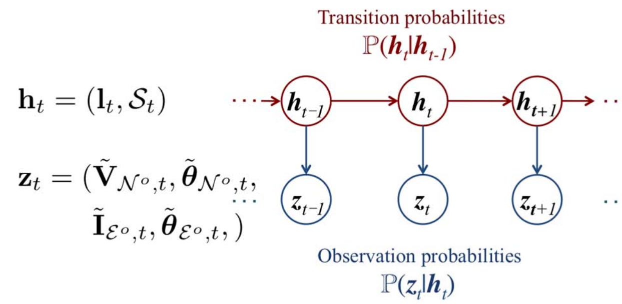

Fig. 2. HMM for line outage detection.

$$\mathcal{I}_{mn} = (\frac{jb_{c,mn}}{2} + y_{mn})\mathcal{V}_m - y_{mn}\mathcal{V}_n \quad (3)$$

$$\mathcal{F}_{mn} = \mathcal{V}_m \mathcal{I}^*_{mn}. \quad (4)$$

B. Hidden Markov Model

An HMM is a statistical model in which the underlying system is assumed to be a Markov process with unobserved (hidden) states. As illustrated in Fig. 2, at time t, the random variable $\mathbf{h}_t$ is the hidden state, and the random variable $\mathbf{z}_t$ is the observation. In this paper, we focus on stationary HMMs, where the hidden states form a stationary Markov process with the state transition probabilities $\mathbb{P}(\mathbf{h}_t|\mathbf{h}_{t-1})$.[1] At time t, conditional on the hidden state, the observation is independent of all other variables and follows the distribution $\mathbb{P}(\mathbf{z}_t|\mathbf{h}_t)$. The transition and observation probabilities together characterize the joint distribution of any sequence of hidden states and observations of the HMM. Given a sequence of observations $\{\mathbf{z}_t : t = 1, \dots, T\}$, one can infer about the sequence of hidden states in the past, namely $\{\mathbf{h}_t : t = 1, \dots, T\}$, and predict the next state or, more generally, a sequence of future observations.

III. System Model

Here, we formulate the problem of line outage detection as an HMM inference problem. We first introduce the components of our HMM.

A. Hidden States and the Transition Probabilities

We denote the line status configuration by a vector $\mathbf{l} \triangleq (l_1, \dots, l_E)$ assuming values in the set $\{0, 1\}^E$, where

[1]When it is clear in the context, we use $\mathbb{P}(X = x)$ to denote the probability mass function for discrete random variables and the probability density function for continuous random variables.

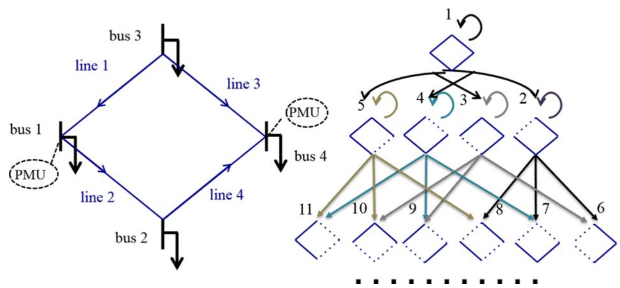

Fig. 3. Example of a transmission network with four buses and four transmission lines and the state transition diagram of the line failure model for this system. We only enumerate the 11 states of line status with at most two line outages and highlight the transitions between configuration which differ in at most one line status.

$l_e = 0$ indicates that line e is disconnected and $l_e = 1$ indicates that line e is connected. Let $\Gamma(\mathbf{l}) = \{e \in \mathcal{E} : l_e = 1\}$ denote the set of good lines for the network configuration $\mathbf{l}$. We denote the line status at time t by $\mathbf{l}_t$ and $\Gamma_t = \Gamma(\mathbf{l}_t)$. The objective is to infer about $\mathbf{l}_t$ over time. We model the line status $\mathbf{l}_t$ and the power injection profile $\mathcal{S}_t$, which are not directly observed, as the hidden state of the HMM as

$$\mathbf{h}_t = (\mathbf{l}_t, \mathcal{S}_t).$$

Given a power injection profile $\mathcal{S}_t$ and a line status configuration $\mathbf{l}_t$, the voltages $\mathcal{V}_t$, the currents $\mathcal{I}_t$ and the power flows $\mathcal{F}_t$ can be solved based on the ac power flow model in (1)–(4). Therefore, we can write $\mathcal{V}_t$, $\mathcal{I}_t$ and $\mathcal{F}_t$ as functions $\mathcal{V}_t(\mathbf{h}_t)$, $\mathcal{I}_t(\mathbf{h}_t)$ and $\mathcal{F}_t(\mathbf{h}_t)$. At time t, if the power flow $\mathcal{F}_{e,t}$ on line e exceeds its thermal limit $\overline{F}_e$, with high probability the line will trip and be disconnected from the grids. When such line trips occur, the power injection profile $\mathcal{S}_t$ changes according to the system regulation rules. Then, based on the new network topology and the possibly new power injection profile, the voltages, currents and the power flows update passively. This process continues until the system stabilizes or a system-wide outage occurs. We model the transition probabilities of the line status as

$$\begin{aligned}\mathbb{P}(\mathbf{h}_{t+1}|\mathbf{h}_t) &= \mathbb{P}(\mathbf{l}_{t+1}|\mathbf{h}_t)\mathbb{P}(\mathcal{S}_{t+1}|\mathbf{l}_{t+1}, \mathcal{S}_t)\\ &= \mathbb{P}(\mathbf{l}_{t+1}|\mathcal{F}_t(\mathbf{h}_t), \mathbf{l}_t)\mathbb{P}(\mathcal{S}_{t+1}|\mathbf{l}_{t+1}, \mathcal{S}_t). \end{aligned} \quad (5)$$

There are two components in the transition probability: $\mathbb{P}(\mathbf{l}_{t+1}|\mathcal{F}_t(\mathbf{h}_t), \mathbf{l}_t)$ is the line failure probability which depends on the power flow profile $\mathcal{F}_t$; $\mathbb{P}(\mathcal{S}_{t+1}|\mathbf{l}_{t+1}, \mathcal{S}_t)$ captures the power redispatch policies[2] and the stochastic nature of the power injections, such as the time-varying generation and consumption.

1) Line Failure Model: We first derive the probabilistic line failure model based on the widely used failure model introduced in [8], [9]. We assume that, once a line fails, it does not recover during the time span we consider. The power system dynamics during the cascading failure is captured by the probabilistic model as follows.

[2]Note that the system is equipped with various automatic control and protection mechanisms in response to disturbances and/or emergencies. Thus, even if the system operator is not aware of the line failures, there could be automatic response on the generations and loads.

When the power flow on a line exceeds its capacity, with high probability the transmission line either breaks down automatically or becomes disconnected from the grids by operator actions. However, in the transient process of cascading failure, overloading a particular line for a short period of time may not necessarily cause it to break down. The line failures can also be caused by other unpredictable events such as bad weather, device malfunction or false line trips. Therefore, considering all of the uncertainties, we model the line outage to be a probabilistic event, which probability is a function of the power flow and the line capacity. In particular, given the power flow $\mathcal{F}_{e,t}$, the event that an operating transmission line fails in the next time slot ($l_{e,t} = 1, l_{e,t+1} = 0$) happens with an independent probability

$$q_{e,t} = \begin{cases} g_1\left(\frac{|\mathcal{F}_{e,t}|}{\overline{F}_e}\right), & \text{if } |\mathcal{F}_{e,t}| < \overline{F}_e \\ g_2\left(\frac{|\mathcal{F}_{e,t}|}{\overline{F}_e}\right), & \text{if } |\mathcal{F}_{e,t}| \geq \overline{F}_e \end{cases} \quad (6)$$

where the initial failure probability $g_1(\cdot)$ and overloading failure probability $g_2(\cdot)$ are nondecreasing functions. Given the power flow profile $\mathcal{F}_t$, the possible line outages are independent events, with probability given by

$$\mathbb{P}(\mathbf{l}_{t+1}|\mathcal{F}_t, \mathbf{l}_t) = \prod_{e \in \Gamma_t \setminus \Gamma_{t+1}} q_{e,t} \prod_{e' \in \Gamma_{t+1}} (1 - q_{e',t}). \quad (7)$$

The transient response of transmission lines is analyzed in [10] in order to justify the above abstract modeling of the line failure process.

Remark 1 (Sparsity): A remarkable feature of the failure model is the sparsity of the transition probabilities $\mathbb{P}(\mathbf{l}_{t+1}|\mathcal{F}_t, \mathbf{l}_t)$, in the sense that, with a moderate system loading profile, only a small portion of the transmission lines are overloaded during the initial stages of the failure process. Moreover, with high probability, the new failures $\Gamma_t \setminus \Gamma_{t+1}$ only occur on a small subset of the overloaded lines. These two facts result in a sparse transition probability $\mathbb{P}(\mathbf{l}_{t+1}|\mathcal{F}_t, \mathbf{l}_t)$. The proposed algorithm exploits this property to reduce the computational complexity.

2) Stochastic Power Injection Policy: On the one hand, the power injection profile $\mathcal{S}_{t+1}$ depends on the system regulation rules. On the other hand, the power injection is stochastic per se due to the time varying supply and demand. These two facts are captured by the conditional probability of $\mathbb{P}(\mathcal{S}_{t+1}|\mathbf{l}_{t+1}, \mathcal{S}_t)$, which can either be learned from the historical data, or derived based on the knowledge of the protection mechanisms in the system.

B. Observation and the Observation Probabilities

There are typically four physical quantities that can be measured in the power system [11]. The conventional SCADA system measures the magnitude of power injection on selected buses and the magnitude of power flow on selected lines; whereas the new technology of PMUs measures the complex voltages on selected buses and the complex currents on selected transmission lines. Throughout this paper, we only consider the PMU measurements, yet the algorithm can also be applied to the setup with other measurements.

At time t, we denote the voltage magnitude at bus n as $\boldsymbol{V}_{n,t}$ and the phase angle at bus n as $\boldsymbol{\theta}_{n,t}$. Similarly, we denote the current magnitude at the line e as $\boldsymbol{I}_{e,t}$ and the phase angle difference at the line e as $\boldsymbol{\theta}_{e,t}$. Denote the subset of buses and the subset of lines where we have the PMU measurements as $\mathcal{N}^o \subseteq \mathcal{N}$ and $\mathcal{E}^o \subseteq \mathcal{E}$. Fig. 3 shows an example of a 4-bus system with PMU measurements. Due to timing misalignment, instrumentation inaccuracy, and modeling uncertainties, the measurements are noisy. We assume the following independent additive noise model of the PMU readings:

$$\tilde{\boldsymbol{V}}_{n,t} = \boldsymbol{V}_{n,t} + \boldsymbol{\epsilon}_{n,t},\ \forall n \in \mathcal{N}^o \tag{8}$$

$$\tilde{\boldsymbol{\theta}}_{n,t} = \boldsymbol{\theta}_{n,t} + \boldsymbol{\varepsilon}_{n,t},\ \forall n \in \mathcal{N}^o \tag{9}$$

$$\tilde{\boldsymbol{I}}_{e,t} = \boldsymbol{I}_{e,t} + \boldsymbol{\eta}_{e,t},\ \forall e \in \mathcal{E}^o \tag{10}$$

$$\tilde{\boldsymbol{\theta}}_{e,t} = \boldsymbol{\theta}_{e,t} + \boldsymbol{\zeta}_{e,t},\ \forall e \in \mathcal{E}^o \tag{11}$$

where $\epsilon_{n,t} \sim \mathcal{N}(0, \sigma_\epsilon^2)$, $\varepsilon_{n,t} \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, $\eta_{e,t} \sim \mathcal{N}(0, \sigma_\eta^2)$ and $\zeta_{e,t} \sim \mathcal{N}(0, \sigma_\zeta^2)$. The observation probability is denoted by $\mathbb{P}(\mathbf{z}_t|\mathbf{h}_t)$ and is given as

$$\mathbb{P}(\mathbf{z}_t|\mathbf{h}_t) = \mathbb{P}(\tilde{\boldsymbol{V}}_{\mathcal{N}^o,t}|\boldsymbol{V}_{\mathcal{N}^o,t})\mathbb{P}(\tilde{\boldsymbol{\theta}}_{\mathcal{N}^o,t}|\boldsymbol{\theta}_{\mathcal{N}^o,t}) \mathbb{P}(\tilde{\boldsymbol{I}}_{\mathcal{E}^o,t}|\boldsymbol{I}_{\mathcal{E}^o,t})\mathbb{P}(\tilde{\boldsymbol{\theta}}_{\mathcal{E}^o,t}|\boldsymbol{\theta}_{\mathcal{E}^o,t}). \tag{12}$$

IV. Line Outage Detection Algorithm

Here, we present our HMM-based dynamic algorithm for the line outage detection problem: estimate the hidden line status state over time based on the given sequence of observations. We first formulate the problem as an inference problem and discuss how the standard forward-backward algorithm can be applied. Then, we show that particle filtering can exploit the specific structure of our model to provide a discrete approximation of the hidden state distribution. We end this section with a discussion of the computational complexity of the approximate algorithm.

A. Problem Formulation

Given an observation sequence $\mathcal{Z}_t = \{\mathbf{z}_1, \ldots, \mathbf{z}_t\}$ at time t, we ask for a probability distribution $\pi_t(\mathbf{h})$ to estimate the hidden state ($\mathbf{h}_t = (\mathbf{l}_t, \mathcal{S}_t)$) distribution

$$\pi_t(\mathbf{h}) \equiv \mathbb{P}(\mathbf{h}_t = \mathbf{h}|\mathcal{Z}_t). \tag{13}$$

These conditional probabilities can then be used as the input of more sophisticated fault diagnosis schemes. For example, we can build threshold-based alarm schemes based on the marginal distribution and report the lines that are most likely to have failed:

$$\pi_t(\mathbf{l}) = \mathbb{E}[\pi_t(\mathbf{l}, \mathcal{S})|\mathbf{l}] = \int_{\mathcal{S}} \pi_t(\mathbf{l}, \mathcal{S}) d\mathcal{S}. \tag{14}$$

Note that the sequence of measurements also reveals more information about how the line status state evolved in the past, which can potentially be used to backtrack the entire realization path of the line failure cascades, identify the initial failure location, and take appropriate actions to recover the faulted lines. In particular, based on the information filtration $\mathcal{Z}_t$, we can update the previous estimation π_τ for all time slots $\tau < t$ to the conditional distribution $\pi_\tau^{(t)}$

$$\pi_\tau^{(t)}(\mathbf{h}) \equiv \mathbb{P}(\mathbf{h}_\tau = \mathbf{h}|\mathcal{Z}_t). \tag{15}$$

Following the Bayesian rules, the conditional probability can be written recursively as

$$\begin{aligned}
\pi_t(\mathbf{h}) &\propto \mathbb{P}(\mathbf{z}_t|\mathbf{h}_t = \mathbf{h})\mathbb{P}(\mathbf{h}_t = \mathbf{h}|\mathcal{Z}_{t-1}) \\
&\propto \mathbb{P}(\mathbf{z}_t|\mathbf{h}_t = \mathbf{h}) \int_{\mathbf{h}'} \mathbb{P}(\mathbf{h}_t = \mathbf{h}|\mathbf{h}_{t-1} = \mathbf{h}')\pi_{t-1}(\mathbf{h}') \\
\pi_t(h) &\propto \mathbb{P}(z_t|h_t = h)\mathbb{P}(h_t = h|z_1, \ldots, z_{t-1}) \\
&\propto \mathbb{P}(z_t|h_t = h) \int_{h'} \mathbb{P}(h_t = h|h_{t-1} = h')\pi_{t-1}(h').
\end{aligned} \tag{16}$$

Similarly, for $\tau < t$, we have

$$\pi_\tau^{(t)}(\mathbf{h}) \propto \underbrace{\mathbb{P}(\mathbf{h}_\tau = \mathbf{h}|\mathcal{Z}_\tau)}_{\pi_\tau(\mathbf{h})} \underbrace{\mathbb{P}(\mathbf{h}_\tau = \mathbf{h}|\mathbf{z}_{\tau+1}, \ldots, \mathbf{z}_t)}_{\rho_\tau^{(t)}(\mathbf{h})} \tag{17}$$

where $\rho_\tau^{(t)}$ can be computed recursively by

$$\rho_\tau^{(t)}(\mathbf{h}) \propto \int_{\mathbf{h}'} \mathbb{P}(\mathbf{h}_{\tau+1} = \mathbf{h}'|\mathbf{h}_\tau = \mathbf{h})\mathbb{P}(\mathbf{z}_{\tau+1}|\mathbf{h}_{\tau+1} = \mathbf{h}')\rho_{\tau+1}^{(t)}(\mathbf{h}'). \tag{18}$$

In the above derivation, we only considered the proportionality and ignored the multiplicative constant, which can be easily determined based on the normalization requirements. Note that this recursion can be done sequentially at the same time scale that we receive the observation sequence.

B. Particle Filter Based Approximation

Next, we present a particle filtering-based approximation algorithm which exploits the structural properties of the model to approximate the distribution π_t and $\pi_\tau^{(t)}$ efficiently.

Recall that the transition probabilities are sparse, namely, there are only a few candidate states which the line status state $\mathbf{l}_t$ transits to with high probability. This motivates us to only maintain a succinct representation of the conditional distributions by approximating the probabilities with discrete "particles". Moreover, by choosing the number of particles, we are able to control the computation complexity. In particular, each particle consists of a sample value and the associated importance weight $(\mathbf{h}_t^i, w_t^i)$, and a distribution π_t is approximated with a set of P particles as

$$\widehat{\pi}_t(\mathbf{h}) = \sum_{i=1}^{P} w_t^i \delta_{\mathbf{h}_t^i}(\mathbf{h}),\ \widehat{\pi}_t(\mathbf{l}) = \sum_{i=1}^{P} w_t^i \delta_{\mathbf{l}_t^i}(\mathbf{l}) \tag{19}$$

where $\delta_{x_0}(x)$ is the Dirac function with the delta mass at x_0. Similarly, the distribution $\pi_\tau^{(t)}$ is approximated by

$$\widehat{\pi}_\tau^{(t)}(\mathbf{h}) = \sum_{i=1}^{P} w_t^i \delta_{\mathbf{h}_\tau^i}(\mathbf{h}),\ \widehat{\pi}_\tau^{(t)}(\mathbf{l}) = \sum_{i=1}^{P} w_t^i \delta_{\mathbf{l}_\tau^i}(\mathbf{l}). \tag{20}$$

In Algorithm 1, we outline the basic flow of the particle filtering based inference algorithm. There are two main steps at each time slot: resampling particles according to the likelihood of the particles, and updating the weights based on the instantaneous observation. As the algorithm runs, some weights may become very small. In order to efficiently use the particles for approximating the distributions, we follow the rule of thumb to resample the particles when the ratio $1/\sum_{i=1}^{P}(w_t^i)^2$ falls below a threshold $P/2$ [12]. The detailed resampling steps are stated in Algorithm 2.

Algorithm 1 Particle Filtering Based Inference Algorithm

Input: Transition probabilities $\mathbb{P}(\mathbf{h}_{t+1}|\mathbf{h}_t)$, observation probabilities $\mathbb{P}(\mathbf{z}_t|\mathbf{h}_t)$, sequential observations $\{\mathbf{z}_t : t = 1, 2, \ldots\}$

Output: Hidden system state probability estimation $\widehat{\pi}_t(\mathbf{h}), \widehat{\pi}_\tau^{(t)}(\mathbf{h}), (\tau < t)$

Initialization: at $t = 0$, draw particles from the stationary distribution: $\mathbf{h}_0^i \sim \mathbb{P}_0(\mathbf{h})$, set weight $w_0^i = 1/P$

for $t = 1, 2 \ldots$ **do**

for $i = 1, 2, \ldots, P$ **do**

Resample the particle according to **(5)**:

$$\mathbf{h}_t^i \sim \mathbb{P}(\mathbf{h}_t|\mathbf{h}_{t-1}^i).$$

Update the particle weight according to **(12)**

$$w_t^i = w_{t-1}^i \mathbb{P}(\mathbf{z}_t|\mathbf{h}_t^i).$$

Optional Particle Management as in Algorithm 2

end for

$$\widehat{\pi}_t(\mathbf{h}) = \sum_{i=1}^{P} w_t^i \delta_{\mathbf{h}_t^i}(\mathbf{h}), \quad \widehat{\pi}_\tau^{(t)}(\mathbf{h}) = \sum_{i=1}^{P} w_t^i \delta_{\mathbf{h}_\tau^i}(\mathbf{h})$$

end for

Algorithm 2 Particle Management

Input: Particles $\{(\mathbf{h}_{1:t}^i, w_t^i) : i = 1, \ldots, P\}$

Output: New particles $\{(\widetilde{\mathbf{h}}_{1:t}^i, \widetilde{w}_t^i) : i = 1, \ldots, P\}$

if $1/\sum_{i=1}^{P}(w_t^i)^2 < P/2$ **then**

for $i = 1, 2, \ldots, P$ **do**

Sample an index j from a multinomial distribution with probabilities $p(j) = w_t^j / \sum_{i=1}^{P} w_t^i$

Set $\widetilde{\mathbf{h}}_{1:t}^i = \mathbf{h}_{1:t}^j$, and $\widetilde{w}_t^i = 1/P$

end for

end if

The following proposition provides the statistical consistency of the algorithm as the number of particles P increases.

Proposition 1: In Algorithm 1, for all t and τ, $\lim_{P\to\infty} \widehat{\pi}_t(\mathbf{h}) = \pi_t(\mathbf{h})$, and $\lim_{P\to\infty} \widehat{\pi}_\tau^{(t)}(\mathbf{h}) = \pi_\tau^{(t)}(\mathbf{h})$ almost surely.

Proof: It is a direct application of the convergence result of Theorem 1 in [13]. We omit the details due to the page limitations. ■

Remark 2 (Outage Detectability): The fact that PMU measurements are not available for all voltages and currents leads to the issue of outage detectability, as there may be more than one line status configurations that produce the same measurements, i.e., $\mathbf{z}_t(\mathbf{h}) = \mathbf{z}_t(\mathbf{h}')$, for $\mathbf{h} \neq \mathbf{h}'$. Conventional algorithms for line outage detection which only use the instantaneous measurement $\mathbf{z}_t$ are unable to distinguish the two states $\mathbf{h}$ and $\mathbf{h}'$. However, the proposed method incorporate the information about the physical dynamics of the cascading failure into the state transition probabilities. It can potentially eliminate some non-distinguishable states by using the sequence of measurements.

C. Complexity Analysis

Consider a frame of T time slots. Given the sequence of measurements $\mathcal{Z}_T = \{\mathbf{z}_1, \ldots, \mathbf{z}_T\}$, we compute the maximum likelihood estimation of the line status realization, i.e., $\mathbf{l}^{(T)} = [\mathbf{l}_1, \ldots, \mathbf{l}_T]$. In particular, by the end of the time frame T, we compute

$$\mathbf{l}^{*(T)} \triangleq \left[\mathbf{l}_t^{*(T)} = \arg\max_{\mathbf{l}} \mathbb{E}\left[\pi_t^{(T)}(\mathbf{l}, \mathcal{S})|\mathbf{l}\right] : t = 1, \ldots, T\right]. \tag{21}$$

There are $\mathcal{O}(2^{ET})$ possible realization paths that the line status state can take. Directly solving a multiway hypothesis testing problem has computational complexity exponential in ET, and quickly becomes intractable as the size of the system and/or the time frame increases. However, with the line failure model we consider, the transition probabilities are sparse, and there are only a few realizations with high probabilities. This fact motivates us to implement the particle filtering based approximation with P particles for $P \ll \mathcal{O}(2^E)$. Then, the hypothesis testing is simplified into a probability estimation problem which can be efficiently and sequentially solved with the proposed method.

Most of the computation comes from the resampling step when computing the transmission probabilities to all $\mathcal{O}(2^E)$ states. However, note that given the power flow profile the lines fail with independent probabilities. Therefore, for each particle, we can execute the resampling step by looking at the transition of every line independently and in parallel. This dramatically reduces the computational complexity to in the order of $\mathcal{O}(PE)$. Also, in order to achieve a target accuracy of line status marginal distribution approximation, we only need to scale the number of particles P linearly in the number of the lines. During the step of weight updating, we need to solve for the ac power flow for each particle, with constant complexity $\mathcal{O}(C)$ depending on the size of the system. In summary, at each time slot, the computation needed for each particle is $\mathcal{O}(E + C)$, and the total computational complexity is in the order of $\mathcal{O}(TP(E + C))$.

On the one hand, the computational complexity scales linearly with the number of the particles P; on the other hand, as a result to law of large number, the accuracy of the discrete approximation for hidden state distribution increases almost in the order of $O(1/\sqrt{P})$. By choosing the number of particles one is able to trade off the computational complexity and the estimation accuracy.

V. Case Study

A. Simulation Setting

We test the proposed HMM-based dynamic line outage detection algorithm on several IEEE standard testing systems: the 6-bus example from page 104–124 in [14], the 4-bus, 14-bus, 30-bus, 57-bus, 118-bus, and 300-bus testing systems from [15].

1) AC Power Flow: To solve the ac power flow (3)–(4), we first specify the slack bus, PV buses, and PQ buses. By convention, one generator bus is chosen to be the slack bus with the fixed and known voltage angle reference and the undetermined power slack. The remaining generator buses are PV buses with given voltage magnitude and real power injection, and the non-generator buses are PQ buses with given active and reactive power demand. We solve the ac power flow equations using the Fast Decoupled method [16] in Matpower [17] with a tolerance level $1e^{-3}$. We ignore all of the generator limits, the branch flow limits, and the voltage magnitude limits.

2) Failure Model Parameters: We adopt the functions $g_1(x) = \delta$ and $g_2(x) = 1 - e^{-\alpha x}$ for line failure probabilities in (6) with parameter $\delta = 0.02$ and $\alpha = 0.1$. We implement the proposed algorithm with the exact parameters ($\delta = 0.02, \alpha = 0.1$) for the line failure model, and also with inaccurate model parameters ($\delta = 0.025, \alpha = 0.2$). By comparing the two cases, we show that the proposed algorithm is robust to model parameter errors. The line capacities are set according to [18].

3) Adaptive Power Injection Policy: The power injection profile is adjusted according to the topology change and is ruled by the following principles.

1) Disconnected components are removed from the system.
2) If the slack bus is disconnected from the system, a PV bus is chosen as the new slack bus.
3) If the original system breaks into several isolated parts, each part is operated separately with its own slack bus.
4) The power injection profile is assumed to be unchanged otherwise.

These principles only aim to capture the basic features of the power injection policies, which have far more complicated details in reality.

4) PMU Observations: The PMU locations are optimized using the method in [19]. We normalize all of the magnitude related readings to in the range $[0, 1]$, and all of the phase related readings to in the range $[-\pi, \pi]$. The measurement noise is modeled as additive Gaussians to the normalized readings, and we set $\sigma_\epsilon = \sigma_\varepsilon = \sigma_\zeta = \sigma_\eta = 0.15$ in (8)–(11). These parameters are also used to compute the observation probabilities in (12).

B. Simulation Result

The proposed algorithm uses 1000 particles to approximate the conditional distribution the line status state $\pi_t(\mathbf{l}) \equiv \mathbb{P}(\mathbf{l}_t = \mathbf{l}|\mathcal{Z}_t)$, and computes the conditional distribution $\pi_t^{(T)}(\mathbf{l}) \equiv \mathbb{P}(\mathbf{h}_t = \mathbf{h}|\mathcal{Z}_T)$ at the end of the time frame $T = 20$. The conventional static algorithm computes the likelihood of the line status states $\mathbb{P}(\mathbf{l}_t|\mathbf{z}_t)$ using only the instantaneous observation $\mathbf{z}_t$.

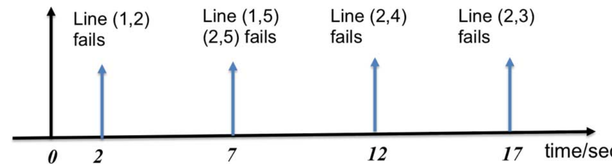

Fig. 4. Realization path of transmission line cascading failure. At time 7, there are two transmission-line outages occurring at the same time. This is intended to demonstrate the ability of our proposed algorithm to recognize multiple line outages at the same time.

Fig. 4 shows one specific line failure realization in the 14-bus system, and Figs. 5 and 6 compare the probabilistic estimates given by the proposed algorithm and the static algorithm. This example sheds light on why the proposed algorithm outperforms the conventional one, and we summarize the main observations below.

1) *Resilience to measurement noise:* The proposed algorithm is able to average the measurement noise over multiple time slots and yield a more accurate estimation of the line status.
2) *Eliminating ambiguous states:* With the limited number of PMUs, two different line status configuration can yield very close or even the same readings. For example, in Fig. 5(a), the first and third states yield statistically indistinguishable instantaneous PMU readings. When the true state is the first state, the static algorithm, which only uses the instantaneous measurements, is not able to distinguish the two states. However, the proposed algorithm, by properly using the information in the previous time slots, can correctly rule out the less probable states and narrow down the estimation to the true state.
3) *Adaptive improvement:* The distribution π_t^T, which is based on measurements in the entire time frame, yields a better estimation over π_t, which is based on the measurements up to time t. This shows that information accumulation over time facilitates identifying of the entire failure path.
4) *Robustness to modeling errors:* The proposed HMM-based inference method is robust to the modeling errors, as Figs. 5 and 6 show similar estimations when the algorithm is implemented with different model parameters.
5) *Recognize simultaneous lines outages:* The proposed algorithm recognizes the two line outages at $t = 7$.

We also study how the computational complexity scales with the size of the system and the number of particles. Fig. 7 compares the computational complexity between the static algorithm (execution time per iteration), and the HMM-based algorithm (average execution time of each particle per iteration). The proposed algorithm can better scale with the size of the system. Moreover, it also shows that the average execution time of each particle per iteration actually decreases with the increasing number of particles used in the algorithm. This is also

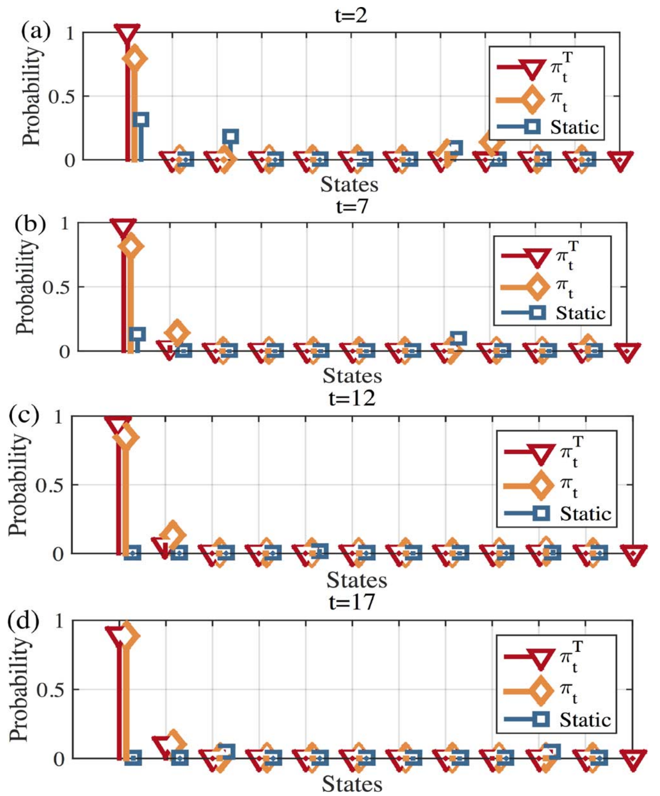

Fig. 5. We compare the estimations of the line status state over the time frame $T = 20$ given by: the static line outage detection algorithm, which computes the conditional distribution $\mathbb{P}(\mathbf{l}_t|\mathbf{z}_t)$; and the proposed HMM-based dynamic line outage detection algorithm, which computes the conditional distributions π_t and π_t^T. In each subfigure, we plot the top 12 states with the highest probabilities given and sorted according to π_1^T. For the HMM-based algorithm, the exact line failure model parameters ($\delta = 0.02, \alpha = 0.1$) are used. Estimation of the line status at (a) $t = 2$, (b) $t = 7$, (c) $t = 12$, and (d) $t = 17$.

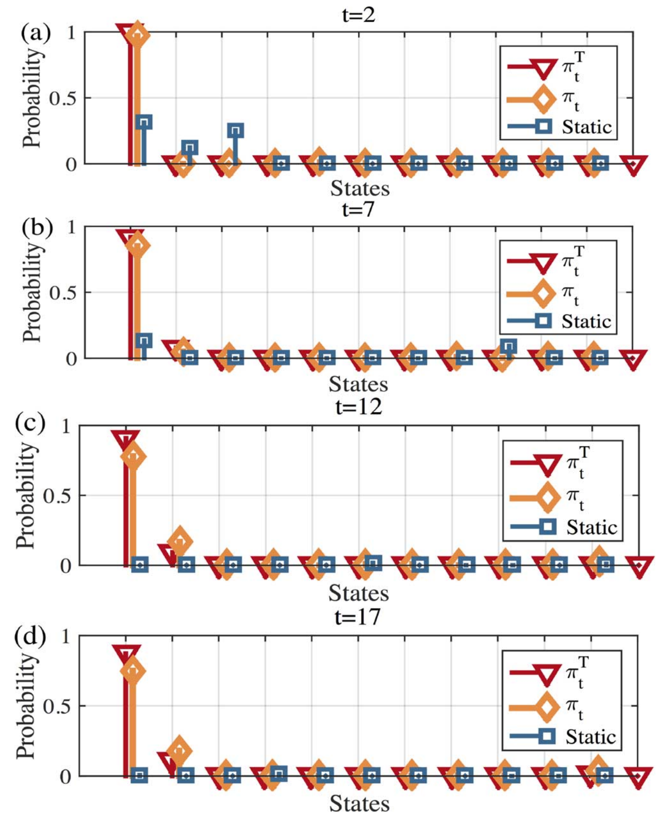

Fig. 6. Same setting as in Fig. 5, except for that misspecified model is used in the inference algorithm. In particular, the line failure model parameters ($\delta = 0.025, \alpha = 0.2$), while the exact parameters are still ($\delta = 0.02, \alpha = 0.1$); the power injection and the line capacity parameter used in the algorithm also have 10 percentage error of the exact value in the simulation. Estimation of the line status at (a) $t = 2$, (b) $t = 7$, (c) $t = 12$, and (d) $t = 17$.

due to the sparsity of the states transitions: many particles are identical and follow the same updating rules.

On the other hand, Fig. 8 shows that, by increasing the number of particles, we can achieve a better estimation accuracy, which is defined in

$$1 - \frac{\|\mathbf{l}^{*(T)} - \mathbf{l}^{(T)}\|_H}{T}, \tag{22}$$

to evaluate the average performance is of the line status state. Therefore, the computational complexity and the estimation accuracy can be balanced by tuning the number of particles P.

C. Average Performance

For a realization in the time frame T and an estimation sequence $\mathbf{l}^{*(T)}$ for the actual hidden state $\mathbf{l}^{(T)}$, we define the accuracy of the estimator to be (22), where $\|\cdot\|_H$ denotes the Hamming distance, which counts the number of positions at which the two sequences differ. We compare the accuracy of the three schemes: the static estimation

$$\mathbf{l}^{*(T)}_{\text{static}} \triangleq [\mathbf{l}^*_{t,\text{static}} = \arg\max \mathbb{P}(\mathbf{z}_t|\mathbf{l}) : t = 1, \ldots, T]$$

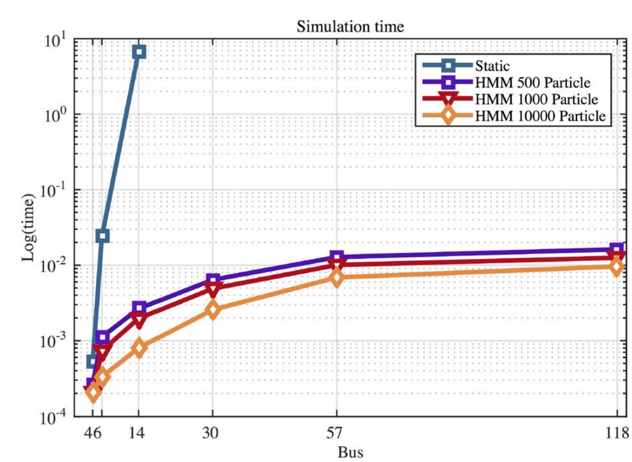

Fig. 7. Comparison between the execution time per iteration of the static algorithm and the average execution time per particle per iteration for different number of particles of the HMM-based algorithm.

and the HMM-based estimations

$$\mathbf{l}^{*(T)}_{\text{hmm realtime}} \triangleq [\mathbf{l}^*_{t,\text{hmm realtime}} = \arg\max \pi_t(\mathbf{l}) : t = 1, \ldots, T]$$
$$\mathbf{l}^{*(T)}_{\text{hmm filtering}} \triangleq [\mathbf{l}^*_{t,\text{hmm filtering}} = \text{argmax}\, \pi_t^{(T)}(\mathbf{l}) : t = 1, \ldots, T].$$

We start all realization paths from state 1, i.e., all lines are in good condition, and terminate each realization path once

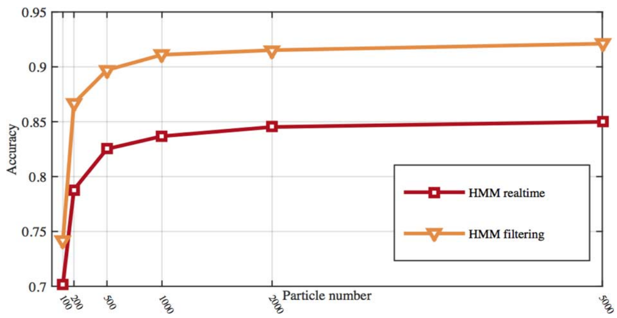

Fig. 8. Accuracy of the proposed algorithm on the 14-bus case for different numbers of particles.

Probability of correctness for different levels of measurement noise			
Noise level ε	0.1	0.15	0.2
Static	0.4094	0.3106	0.2231
HMM Real Time	0.909	0.789	0.687
HMM With Filter	0.9936	0.9588	0.9135

Fig. 9. Average performance of line status estimation given by the three algorithms. We choose three noise levels for $\sigma_\epsilon = \sigma_\varepsilon = \sigma_\zeta = \sigma_\eta$ to be 0.1, 0.15, and 0.2. The HMM-based algorithms outperform the static algorithm for all noise levels.

it reaches an absorbing state. Here a state is an absorbing state if there is no feasible power solution under the current power injection and network configuration. We simulated 2000 realization paths for three different levels of the measurement noise. The table in Fig. 9 compares the average performance of the three schemes, and we can see that for every noise level, the proposed algorithm significantly outperforms the static algorithm.

VI. Concluding Remarks

In this paper, we proposed an HMM-based algorithm to detect line failures in transmission networks. The algorithm leverages our understanding of the dynamics of cascading failure in the system.

Although in this paper, we focus on transmission line outage detection. The proposed HMM and particle filter methods can be applied to identify other faults in power systems. For example, detecting components failures (relays, breakers, transformers) in the grids; detecting faults in distribution network where not every line is closely monitored; detecting unexpected load fluctuations and generator failures in transmission networks.

Future works include: 1) rigorously quantifying the performance improvement in terms of outage detectability and estimation accuracy; 2) more sophisticated implementations of particle filtering to speed up the inference algorithm; 3) analyzing the system with more realistic power injection policy; and 4) more case studies using real power system data.

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