HARVARD
John A. Paulson
School of Engineering
and Applied Sciences

Scalable Reinforcement Learning for Multi-Agent Networked Systems

Guannan Qu[1,2] and **Na Li**[1]

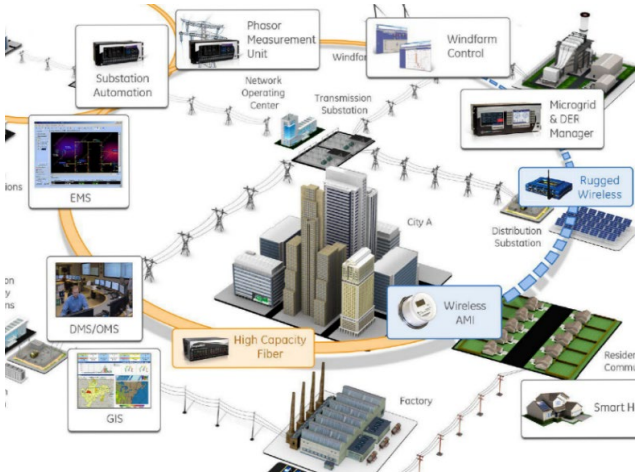
Harvard University

California Institute of Technology

CDC 2019

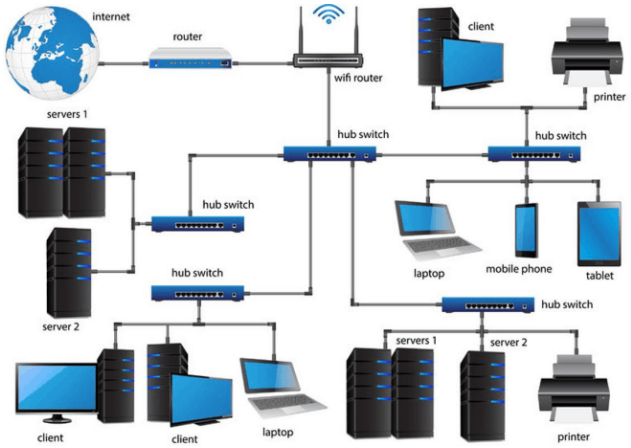
Control of Large Scale Networks

Traffic



Energy

Epidemic/opinion…



Computer Network

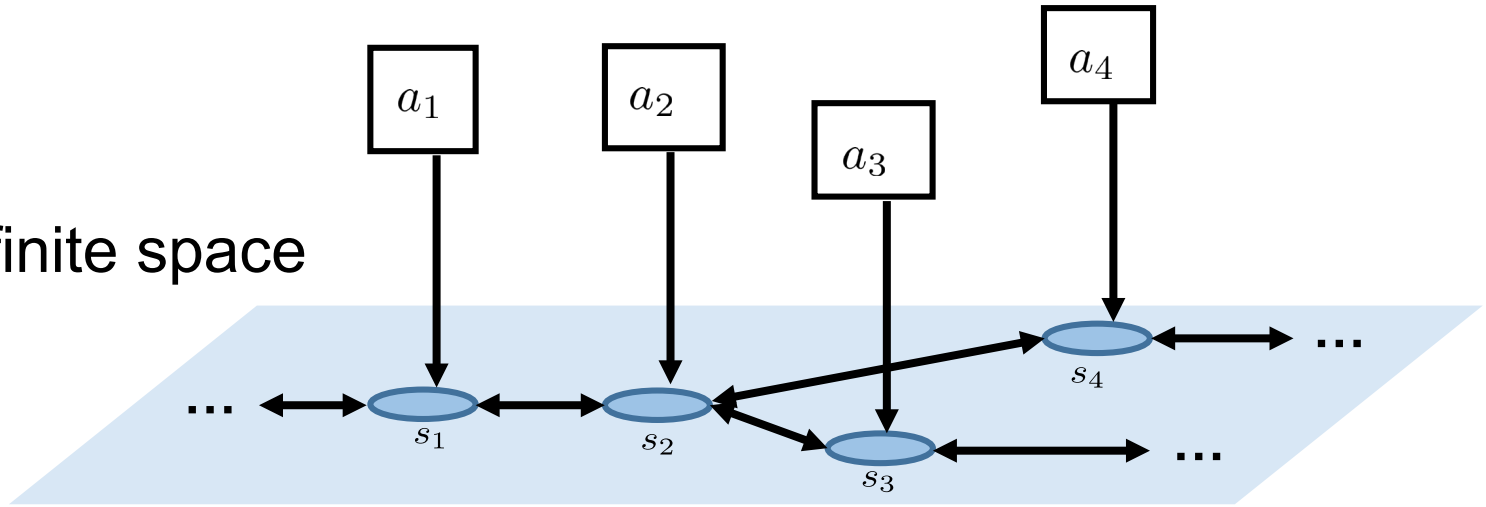
- Large scale ➡ Tractable/Efficient methods?

- Model assumption not correct
- Parameters not correct
- Large data available

} ➡ Reinforcement Learning

Markov Decision Process (MDP) over Networks

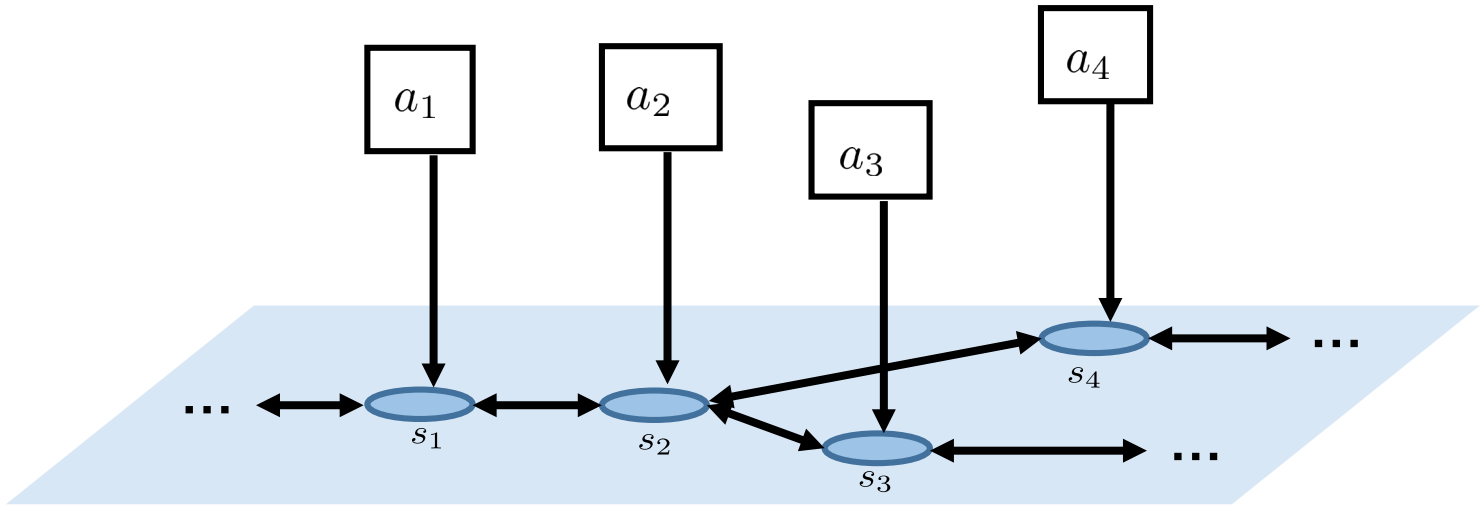
a_i, s_i: finite space



State $s = (s_1, \ldots, s_n) \in S_1 \times S_2 \times \cdots \times S_n := \mathcal{S}$

Action $a = (a_1, \ldots, a_n) \in \mathcal{A}_1 \times \mathcal{A}_2 \times \cdots \times \mathcal{A}_n := \mathcal{A}$

Markov Decision Process over Networks



State Transition $$P(s(t+1)|s(t),a(t)) = \prod_{i=1}^{n} P_i(s_i(t+1)|s_{N_i}(t),a_i(t)).$$

i's next state

Own state and Neighbor's state

Own action

Stage Reward $$r(s) = \sum_{i=1}^{n} r_i(s_i,a_i)$$

Objective $$\text{Max} \quad \lim_{T\to\infty} \mathbb{E}\left[\frac{1}{T}\sum_{t=0}^{T} r(s(t),a(t))|s(0)=s\right]$$

Challenges

Challenge 1 **State/Action Space Exponentially Large!**

$$|\mathcal{S}| = \prod_{i=1}^{n} |\mathcal{S}_i| \qquad |\mathcal{A}| = \prod_{i=1}^{n} |\mathcal{A}_i|$$

Methods for centralized MDP/RL

- Value/Policy Iteration, Q-learning [Watkins1989]
- Actor-Critic [Konda 2000]
- Linear Programming [Bertsekas 1976]

} **Time/space complexity exponential in *n***

Methods in Multi-Agent MDP/RL to deal with scalability

- Independent Learners [Claus and Boutilier, 1998]
- Linear function approximation [Zhang 2018]
- Neuro Networks [Lowe et al., 2017]

} **unclear how to choose approximator that guarantee (near)-optimality**

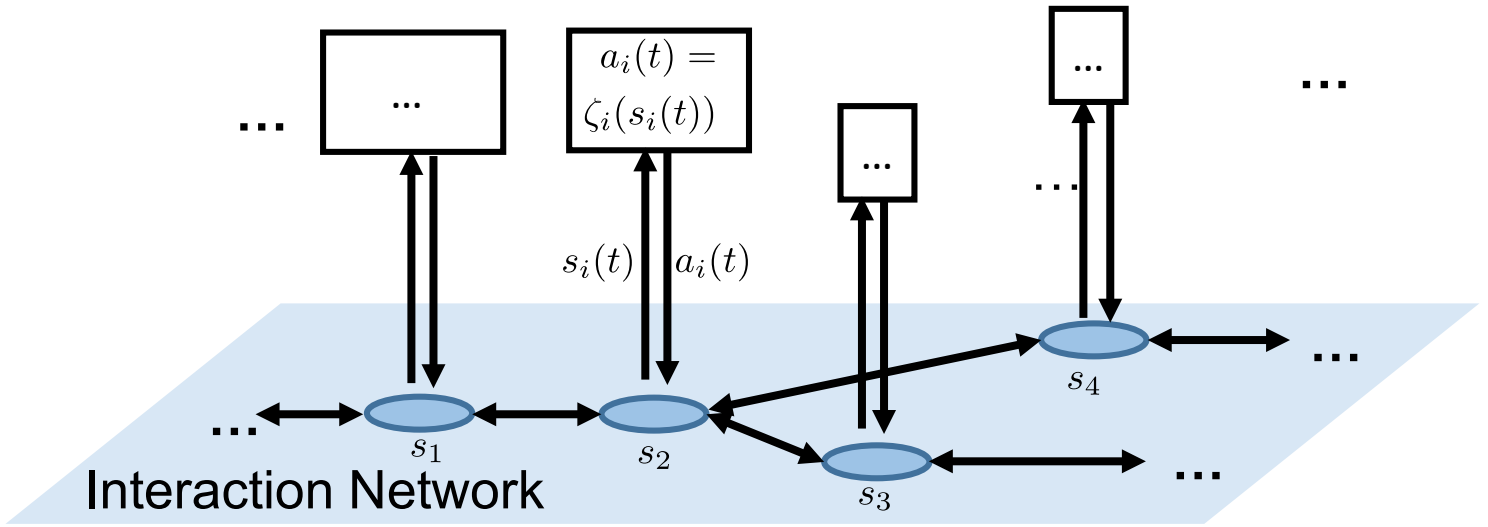
Computation Complexity Results

Blondel and Tsitsiklis, 2000; Whittle, 1988; Papadimitriou and Tsitsiklis, 1999

Challenges

Challenge 2: **Information constraint**

Local policy (deterministic): $a_i(t) = \zeta_i(s_i(t))$ where $\zeta_i : \mathcal{S}_i \to \mathcal{A}_i$



This Talk

Is it possible to exploit network structure to design scalable algorithms that find a (near)-optimal localized policy?

Network structure (Local dependence) → **Efficient methods!**

Key: exponential decay property

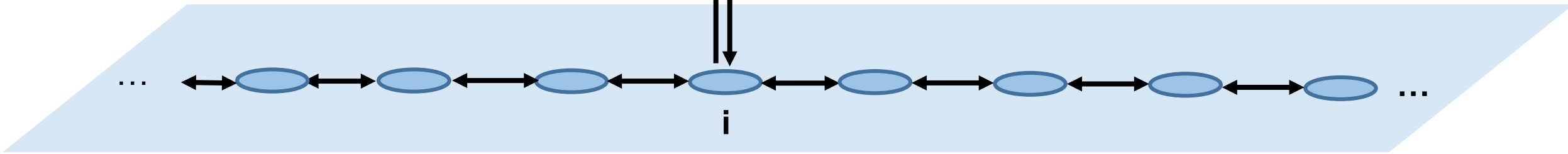
- **assume model is known (this CDC paper)**
- model-free reinforcement learning (latest work)

Decomposition of Average Reward

Localized Policy $a_i(t) = \zeta_i(s_i(t))$

Joint Policies

$\zeta = (\zeta_1, \ldots, \zeta_n)$

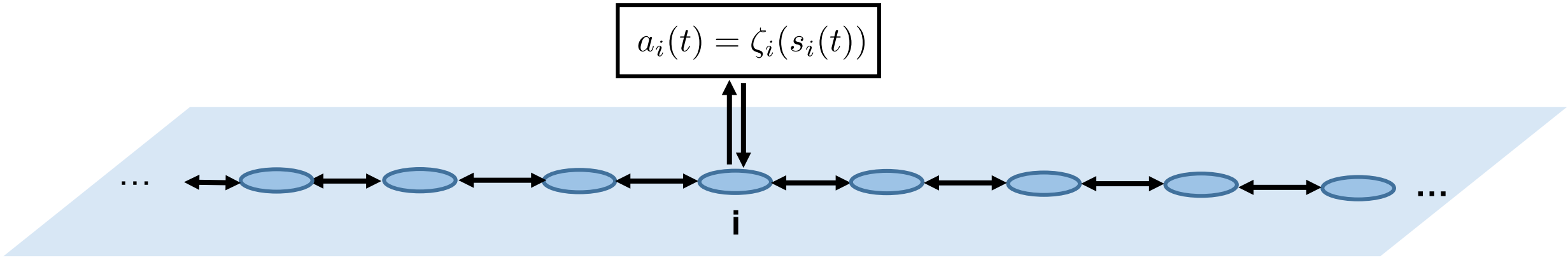


Average Reward: $R(\zeta) = \mathop{\mathbb{E}}_{s\sim\pi(\zeta)} r(s,a) = \sum_{i=1}^{n} \mathop{\mathbb{E}}_{(s_i,a_i)\sim\pi_i(\zeta)} r_i(s_i,a_i) := \sum_{i=1}^{n} R_i(\underbrace{\zeta_1, \ldots, \zeta_n}_{})$

Depends on policies of all nodes

$\pi_i(\zeta)$ Marginalized stationary distribution at node i

$R_i(\zeta)$ Expected reward at node i in stationarity



$$\max \quad R(\zeta) = \sum_{i=1}^{n} R_i(\underbrace{\zeta_1, \ldots, \zeta_n}_{\text{Exponentially many combinations}})$$

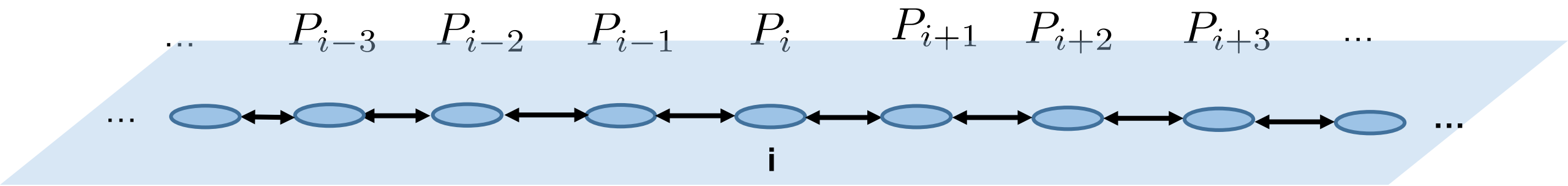
Outline of our approach:

- **Approximate R_i using a much simpler function**
- Efficient alg. to optimize policies based on the approximate
- Analyze error between the approximate and true reward

Approximation of R_i

Local Transition Structure

$$P(s(t+1)|s(t),a(t)) = \prod_{i=1}^{n} P_i(s_i(t+1)|s_{N_i}(t),a_i(t)).$$



$a_i(t) = \zeta_i(s_i(t))$ Localized Policies

$\zeta = (\zeta_1, \ldots, \zeta_n)$ Joint Policies

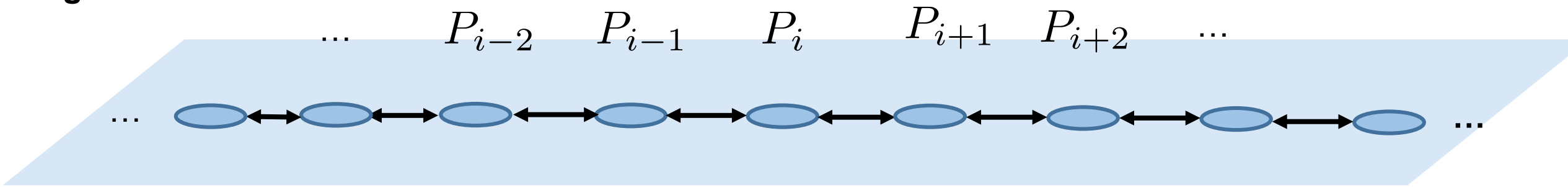
$\pi_i(\zeta)$ Marginalized stationary dist. at node i

$R_i(\zeta)$ Expected reward at node i in stationarity

$$R_i(\zeta) = \mathop{\mathbb{E}}_{(s_i,a_i)\sim\pi_i(\zeta)} r_i(s_i,a_i)$$

Original MDP

Local transition probabilities



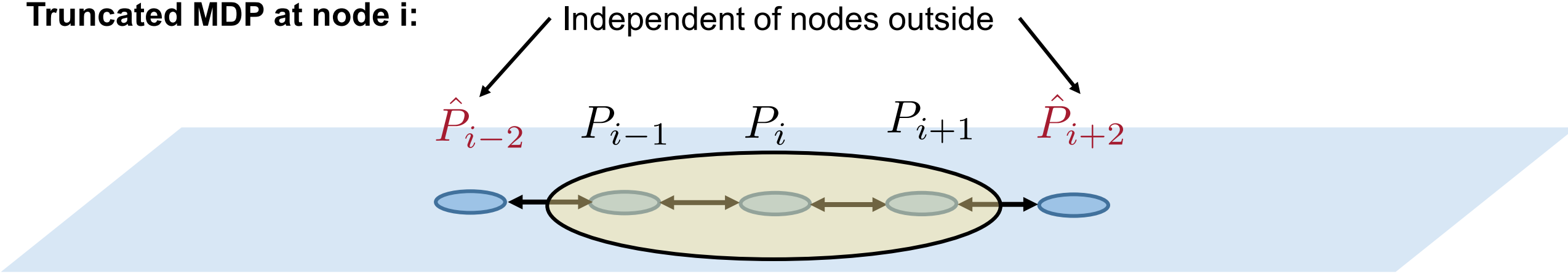
$\pi_i(\zeta)$ Marginalized stationary dist. at node i

$R_i(\zeta)$ Expected reward at node i in stationarity

$\hat{\pi}_i\left(\zeta_{N_i^k}\right)$ Marginalized stationary dist. for **truncated** model

$\hat{R}_i\left(\zeta_{N_i^k}\right)$ **Approximate expected reward at i**

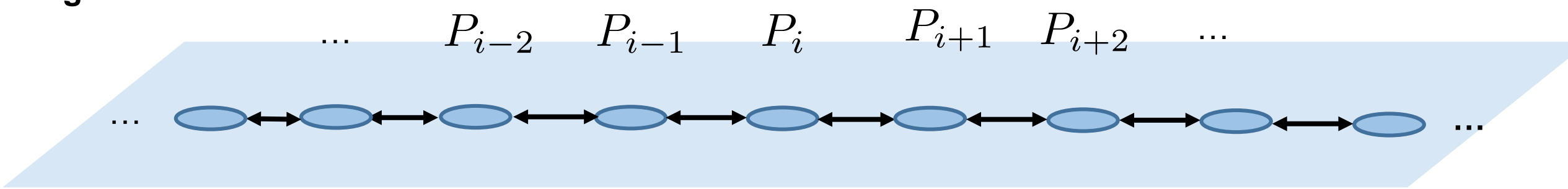
Truncated MDP at node i:



k-hop neighborhood (with k=1)

Original MDP

Local transition probabilities



$\pi_i(\zeta)$ Marginalized stationary dist. at node i

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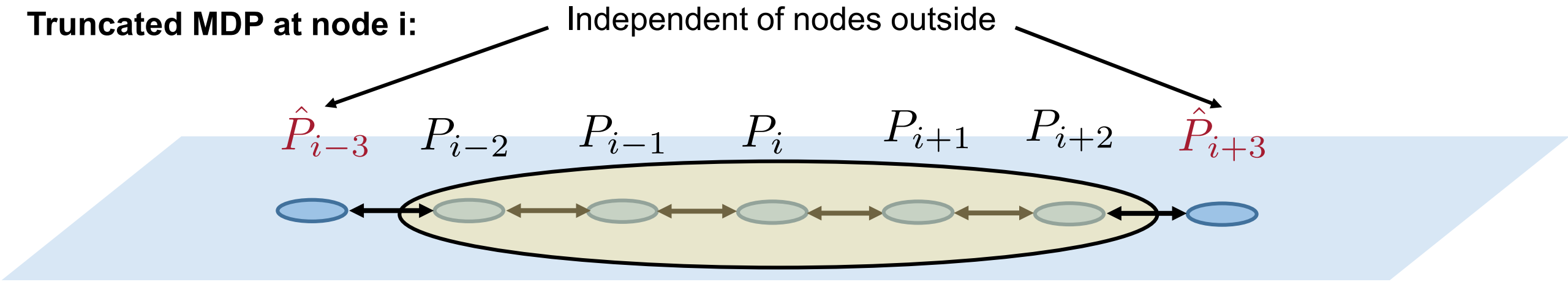
$\hat{\pi}_i\left(\zeta_{N_i^k}\right)$ Marginalized stationary dist. for **truncated** model

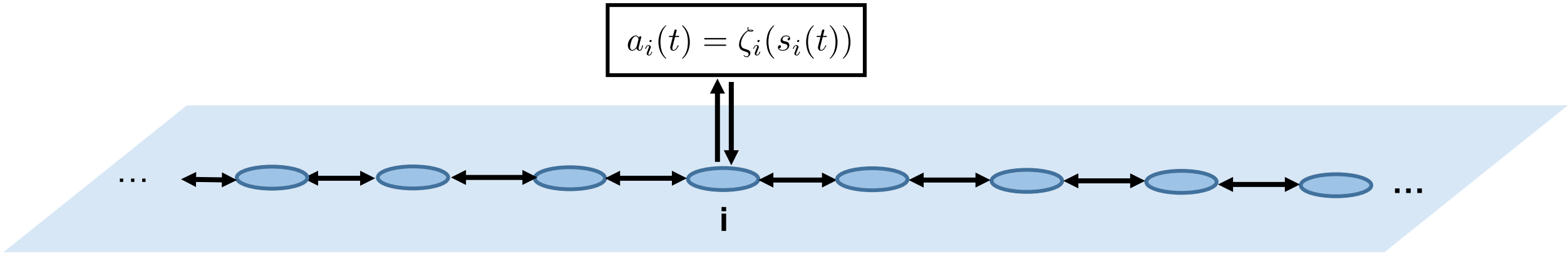
$\hat{R}_i\left(\zeta_{N_i^k}\right)$ **Approximate expected reward at i**

Truncated MDP at node i:

Independent of nodes outside



k-hop neighborhood (with k=2)

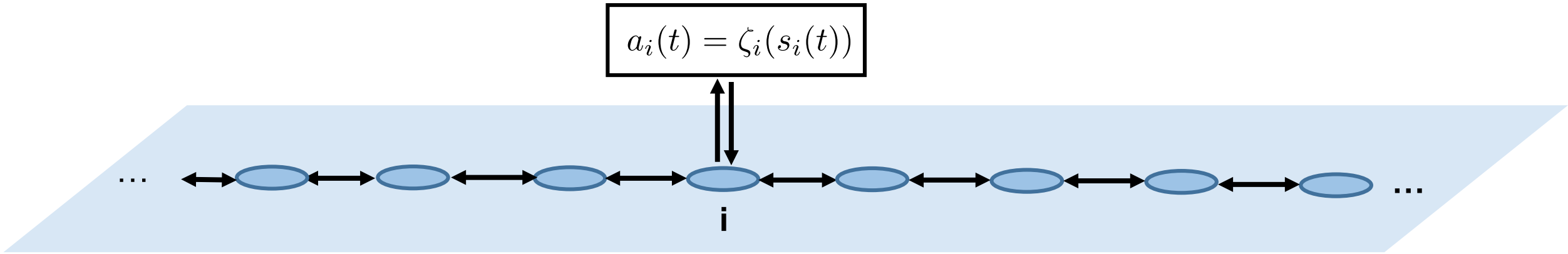


$$\max \quad R(\zeta) = \sum_{i=1}^{n} R_i(\zeta_1, \ldots, \zeta_n)$$

Outline of our approach:

Much smaller combinations

- **Approximate R_i using a much simpler function** $\max \hat{R}(\zeta) = \sum_{i=1}^{n} \hat{R}_i(\zeta_{N_i^k})$
- Efficient alg. to optimize policies based on the approximate
- Analyze error between the approximate and true reward



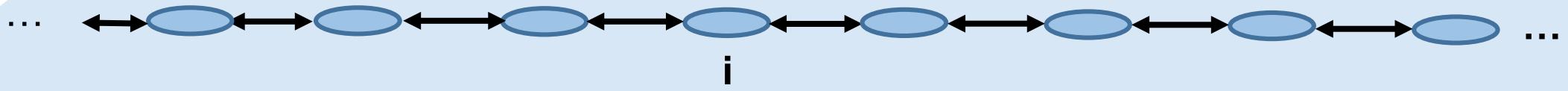
$$\max \quad R(\zeta) = \sum_{i=1}^{n} R_i(\zeta_1, \ldots, \zeta_n)$$

Outline of our approach:

Much smaller combinations

- Approximate R_i using a much simpler function $\max \hat{R}(\zeta) = \sum_{i=1}^{n} \hat{R}_i(\zeta_{N_i^k})$
- **Efficient alg. to optimize policies based on the approximate**
- Analyze error between the approximate and true reward

Efficient Alg for $\max_{\zeta_i} \sum \hat{R}_i(\zeta_{N_i^k})$: Dynamic Programming



The case of a line, and k=1 hop neighbor truncation,

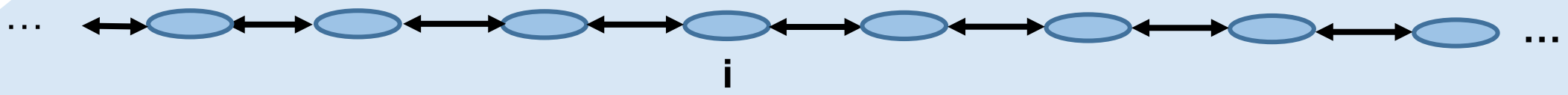
$$\max_{\zeta} \hat{R}(\zeta) = \max_{\zeta_1,\ldots,\zeta_n} \hat{R}_1(\zeta_1,\zeta_2) + \ldots + \hat{R}_i(\zeta_{i-1},\zeta_i,\zeta_{i+1}) \ldots + \hat{R}_n(\zeta_{n-1},\zeta_n)$$

$$= \max_{\zeta_1,\ldots,\zeta_i} \hat{R}_1(\zeta_1,\zeta_2) + \ldots \hat{R}_{i-1}(\zeta_{i-2},\zeta_{i-1},\zeta_i) + \underbrace{\max_{\zeta_{i+1},\ldots,\zeta_n} \hat{R}_i(\zeta_{i-1},\zeta_i,\zeta_{i+1}) \ldots + \hat{R}_n(\zeta_{n-1},\zeta_n)}_{V_i(\zeta_{i-1},\zeta_i)}$$

Properties: $V_i(\zeta_{i-1},\zeta_i) = \max_{\zeta_{i+1}} \hat{R}_i(\zeta_{i-1},\zeta_i,\zeta_{i+1}) + V_{i+1}(\zeta_i,\zeta_{i+1})$

$$\hat{R}(\zeta) = \hat{R}_1(\zeta_1,\zeta_2) + V_2(\zeta_1,\zeta_2)$$

Efficient Alg for $\max_{\zeta_i} \sum \hat{R}_i(\zeta_{N_i^k})$: Dynamic Programming



The case of a line, and k=1 hop neighbor truncation,

Dynamic Programming

Backward Sweep

$$V_i(\zeta_{i-1}, \zeta_i) = \max_{\zeta_{i+1}} \hat{R}_i(\zeta_{i-1}, \zeta_i, \zeta_{i+1}) + V_{i+1}(\zeta_i, \zeta_{i+1})$$

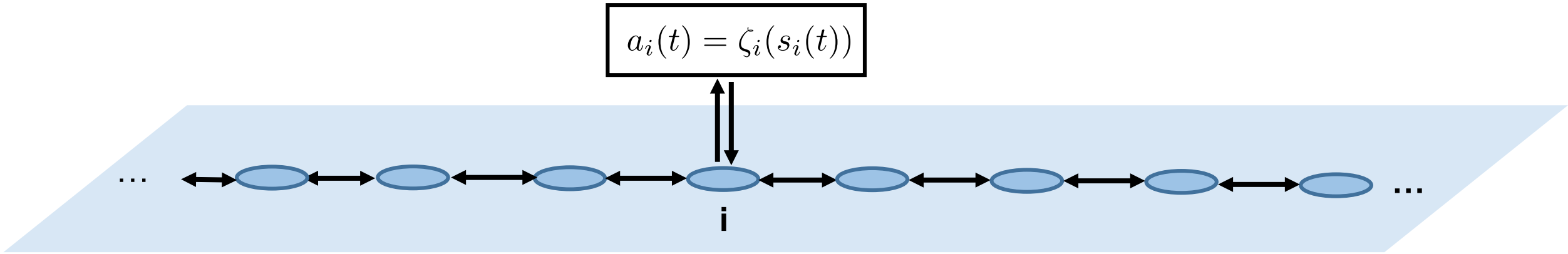
Forward Sweep

$$(\zeta_1^*, \zeta_2^*) = \arg\max_{\zeta_1, \zeta_2} \hat{R}_1(\zeta_1, \zeta_2) + V_2(\zeta_1, \zeta_2)$$

$$\zeta_{i+1}^* = \arg\max_{\zeta_{i+1}} \hat{R}_i(\zeta_{i-1}^*, \zeta_i^*, \zeta_{i+1}) + V_{i+1}(\zeta_i^*, \zeta_{i+1})$$

Proposition (Informal) When the graph is tree, this method finds a maximizer $\zeta^* = (\zeta_1^*, \ldots, \zeta_n^*)$ of $\hat{R}(\zeta_1, \ldots, \zeta_n)$ within time scaling in the policy space size of the largest k-hop neighborhood.

Comparison: much more efficient than directly optimize R!

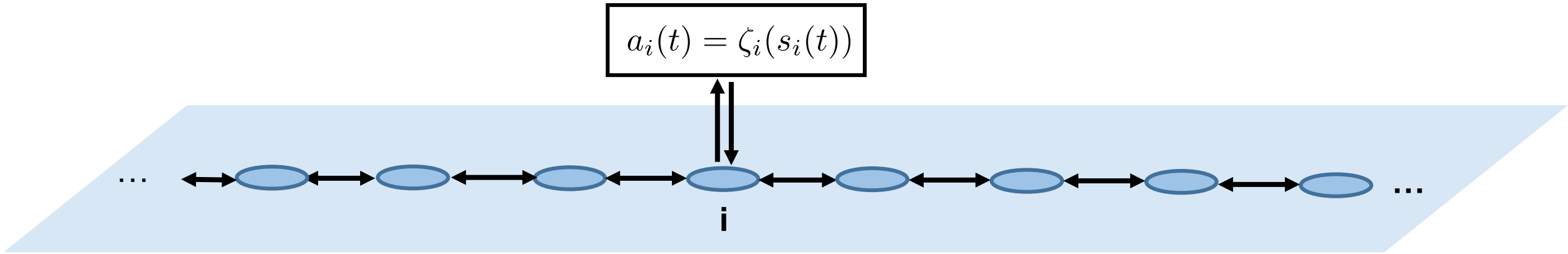


$$\max \quad R(\zeta) = \sum_{i=1}^{n} R_i(\zeta_1, \ldots, \zeta_n)$$

Outline of our approach:

Much smaller combinations

- Approximate R_i using a much simpler function: $\max \hat{R}(\zeta) = \sum_{i=1}^{n} \hat{R}_i(\zeta_{N_i^k})$
- **Efficient alg. to optimize the approximate reward: Dynamic Programming**
- Error between the approximate and true reward



$$\max \quad R(\zeta) = \sum_{i=1}^{n} R_i(\zeta_1, \ldots, \zeta_n)$$

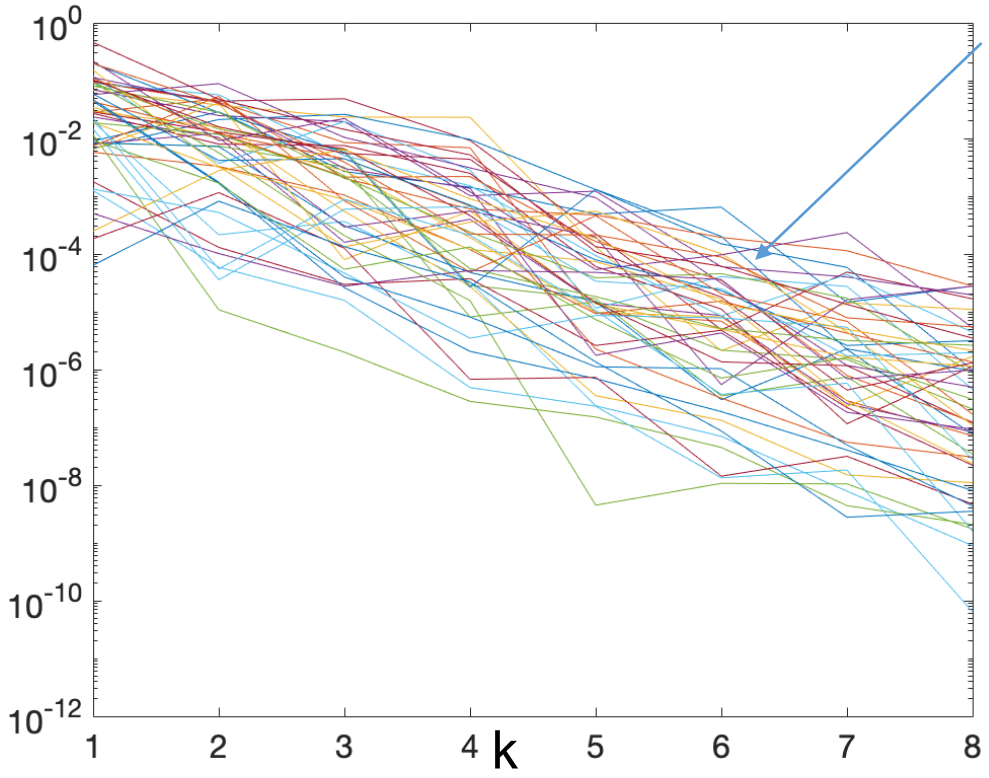
Outline of our approach:

Much smaller combinations

- Approximate R_i using a much simpler function: $\max \hat{R}(\zeta) = \sum_{i=1}^{n} \hat{R}_i(\overbrace{\zeta_{N_i^k}}^{})$
- Efficient alg. to optimize the approximate reward: Dynamic Programming
- **Error between the approximate and true reward**

Error decays exponentially in k

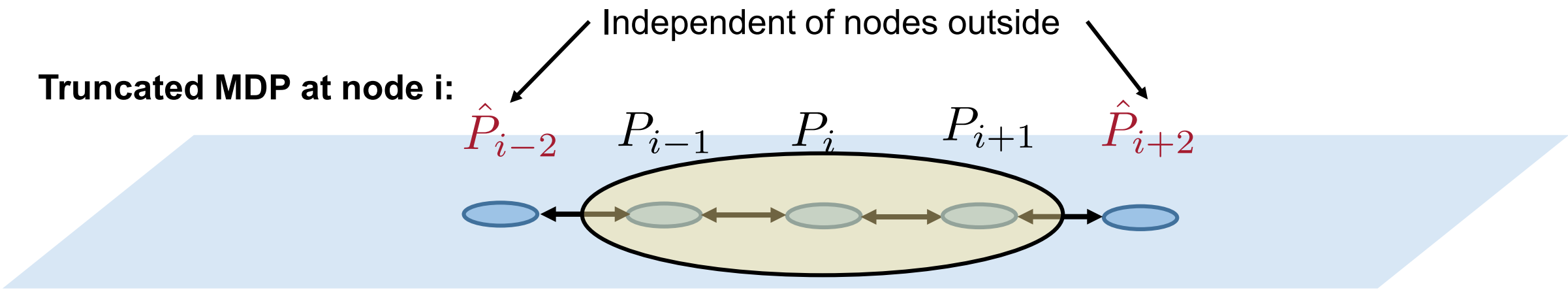
Error between true and approximate reward



Doing truncation might not have a big error, even for small k

Exponential Decaying Property: Formal Definition

Truncated MDP at node i:



$\pi_i(\zeta)$ Marginalized Stationary Distribution at node i of the full model

$\hat{\pi}_i(\zeta_{N_i^k})$ Marginalized Stationary Distribution at node i of the truncated model

(c, ρ)-exponential decay holds if for all i, all policy $\zeta = (\zeta_1, \ldots, \zeta_n)$

$$TV(\pi_i(\zeta), \hat{\pi}_i(\zeta_{N_i^k})) \leq c\rho^{k+1}$$

for some constant $c > 0$, $\rho \in (0, 1)$

Conditions for Exponential Decaying Property to Hold: Example

Define interaction strength matrix $C = [C_{ij}]$

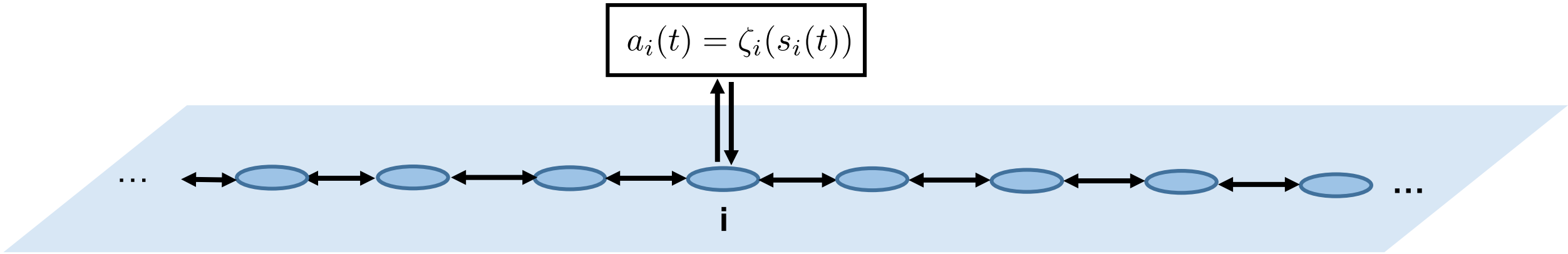
$$C_{ij} = \begin{cases} 0 & \text{if } j \notin N_i \\ \sup_{s_{N_i/j}, a_i} \sup_{s_j, s_j'} TV(P_i(\cdot|s_j, s_{N_i/j}, a_i), P_i(\cdot|s_j', s_{N_i/j}, a_i)) & \text{if } j \in N_i, j \neq i \\ \sup_{s_{N_i/j}} \sup_{s_i, s_i', a_i, a_i'} TV(P_i(\cdot|s_i, s_{N_i/i}, a_i), P_i(\cdot|s_i', s_{N_i/i}, a_i')) & \text{if } j = i \end{cases}$$

Lemma If $\|C\|_1 \leq \rho < 1$, then the $(\frac{1}{1-\rho}, \rho)$-exponential decaying property holds, i.e.

$$TV(\pi_i(\zeta), \hat{\pi}_i(\zeta_{N_i^k})) \leq \frac{1}{1-\rho}\rho^{k+1}$$

for all i, all policy $\zeta = (\zeta_1, \ldots, \zeta_n)$.

Note: For MDP with discounting rewards ($R = \sum_t \gamma^t r(t)$), exponential decaying property naturally holds under the ergodic condition (Qu, Weirman, Li, 2019).



$$\max \quad R(\zeta) = \sum_{i=1}^{n} R_i(\zeta_1, \ldots, \zeta_n)$$

Summary:

Much smaller combinations

- Approximate R_i using a much simpler function: $\max \hat{R}(\zeta) = \sum_{i=1}^{n} \hat{R}_i(\zeta_{N_i^k})$
- Efficient alg. to optimize the approximate reward: Dynamic Programming
- **Error between the approximate and true reward:** Exponential decaying ρ^k !

What if we do not know the model parameters?

Review: Policy Gradient in the full information case (one agent)

Parameterized Policy: $a(t) = \zeta^{\theta}(s(t))$

Q Function: $Q^{\theta}(s,a) = \lim_{T\to\infty} \mathbb{E}^{\theta}\left[\frac{1}{T}\sum_{t=0}^{T} r(s(t),a(t)) \middle| s(0)=s, a(0)=a\right]$ Limiting Average reward

$Q^{\theta}(s,a) = \mathbb{E}^{\theta}\left[\sum_{t=0}^{\infty} \gamma^t r(s(t),a(t)) \middle| s(0)=s, a(0)=a\right]$ Discounting reward

Policy Gradient Theorem [Sutton 2000]

$$\nabla R(\theta) = \mathbb{E}_{s\sim\pi^{\theta}, a\sim\zeta^{\theta}(\cdot|s)} Q^{\theta}(s,a) \nabla \log \zeta^{\theta}(a|s).$$

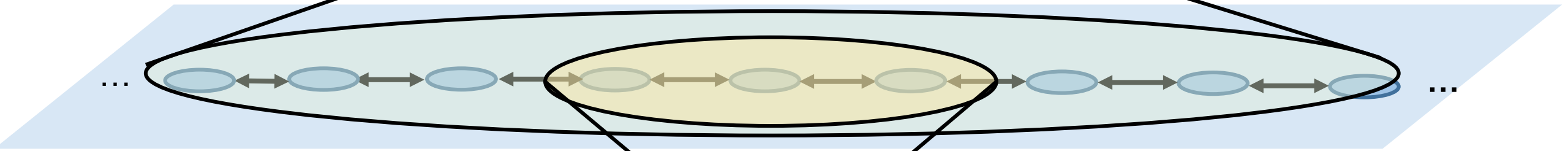
Actor-Critic Methods [Konda 2000]

Actor: $\theta \leftarrow \theta + \eta_t \times \nabla R(\theta)$

Critic: $Q(s(t-1),a(t-1)) \leftarrow Q(s(t-1),a(t-1)) + \alpha_t \times$ (TD Error)

Full Q function: $Q^\theta(s,a) = \sum_i^n Q_i(s,a)$

$Q_i(s_1, \ldots, s_n, a_1, \ldots, a_n)$ Exponentially large table



$\hat{Q}_i(s_{N_i^k}, a_{N_i^k})$ Only depends on k-hop neighborhood

"Truncated" Q function.

Scalable Actor-Critic (SAC) for Networked MDP Learning

Full Q function $Q_i(s_1, \ldots, s_n, a_1, \ldots, a_n)$

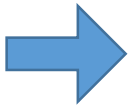
Full Policy Gradient $\nabla_{\theta_i} J(\theta) = \mathbb{E}_{s\sim\pi^\theta, a\sim\zeta^\theta(a|s)} \nabla_{\theta_i} \log \zeta_i^{\theta_i}(a_i|s_i) Q^\theta(s,a)$

Truncated Q Function $\hat{Q}_i(s_{N_i^k}, a_{N_i^k})$

Truncated Policy Gradient $h_i(\theta) = \mathbb{E}_{s\sim\pi^\theta, a\sim\zeta^\theta(a|s)} \nabla_{\theta_i} \log \zeta_i^{\theta_i}(a_i|s_i) \sum_{j\in N_i^k} \hat{Q}_j^\theta(s_{N_j^k}, a_{N_j^k})$

Lemma (informal) if the exponential decay property holds, then

$\|h_i(\theta) - \nabla_{\theta_i} J(\theta)\| \leq O(\rho^k)$ very small error even for small k

Truncated Q + Truncated Policy Gradient  Scalable RL for Networked Systems

Latest work: Guannan Qu, Adam Wierman and Na Li, Scalable Reinforcement Learning of Localized Policies for Multi-Agent Networked Systems, arXiv:1912.02906.

Summary

Exploit network structure to design scalable algorithms that find a (near)-optimal localized policy

Network structure (Local dependence) → **Efficient methods!**

Key: exponential decay property

- **Assume model is known (This CDC paper)**
  - Truncation
  - Exponential Decay Property
- Model-free reinforcement learning (latest work)
  - Truncated Q function
  - Scalable Actor-Critic (SAC)

Thank you!