Coordinating automatic demand response with small amount of communication

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***Abstract*—Demand response (DR) is essential for smart grid. Recent DR programs always require iterative two way communication to find the efficient DR operating point. Because of the reliability and limited resources of communication, reducing communication amount of DR is important for DR development. This paper propose DR algorithms where only one-way communication is needed. Furthermore, each iteration is able to be reduced to single-bit.**

I. Introduction

II. Problem Formulation

Consider a power distribution network with a set N of customers/users that are served by one utility company. Associated with each customer $i \in N$ is its power load q_i which is assume to be within a limit $[\underline{q}_i, \bar{q}_i]$. The demand response we consider is to coordinate all the users' consumption such that $\sum_i q_i \leq Q$. One interpretation of Q is the feeder (transformer) limit. Other scenarios is that there might be supply deficit due to a generator failure or decrease of renewable generation. Assume that each customer i attains $U_i(q_i)$ when its power draw is q_i, which is assumed to be a concave and differentiable function. We assume that the objective of DR is to coordinate the users' power consumption such that the total electricity Q is allocated in an efficient and fair way, which is formulated as the following optimization problem,

$$\max_{q_i, i\in N} \quad \sum_i U_i(q_i) \tag{1a}$$

$$\text{s.t.} \quad \underline{q}_i \leq q_i \leq \bar{q}_i \tag{1b}$$

$$\sum_i q_i \leq Q_i \tag{1c}$$

As discussed in Section I, we would like to answer the question whether we can reduce the communication requirement by

Note that this model abstract away the network constraints. The purpose of this paper is to study whether we can limited communication in coordinating demand response. Thus we pick the simpest model to illustrate our idea. The future work will require consider the network constraints in details. One the other hand, when we consider distribution network DR problem, this model is still meaningful because limited the load consumption will only make the network less congested. Thus the network constraints (e.g., voltage constraints, line thermal limit constraints) are not as important as the feeder constraint (1c).

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III. Demand response with two-way iterative communication

There exists a large amount of work proposing distributed algorithms to coordinate the users to solve the above problem ([**?**]). Typically, it will require the two-way iterative communication between the utility company and users.

- Initially, the utility company randomly chooses a price $p(0)$ and annouce the price information to each user. Set $k = 0$.
- Upon receiving price $p(k)$ announced by the utility company, each customer i calculate its power consumption $q_i(k)$
$$q_i(k) = [(U_i')^{-1}(p(k))]^+, \tag{2}$$
and submits it to the utility company through the communication link.
- Upon gathering the power consumption request $q_i(k)$ from customers, the utility company updates the price according to
$$p(k+1) = [p(k) - \gamma(\sum_i q_i(k) - Q)]^+, \tag{3}$$
and announces price $p(k+1)$ to customers.
- let $k+1 \leftarrow k$ and repeat from step 2.

The algorithm is a dual gradient algorithm of problem 1. Using the standard dual gradient algorithm analysis [], we know that the algorithm converges to the optimal solution, as described in the following theorem.

Theorem 1 *Algorithm converges to optimal solution of if the stepsize* $\gamma \leq$ *....*

IV. Demand response with one-way and single-bit communication

A. Implementation of Algorithm 1 using one-way communication

Though the algorithm 1 literally requires two-way communication where utility company sends price to users and users send the power consumption request to utility company, the implementation can be done using one-way communication because the users do not need to use communication to send the power consumption to the utility company. Note that the algorithm only requires the aggregated the power consumption of users each step. Therefore, at each step, instead of calculating the aggregated power consumption using the information passed from users, the utility company can just measure the power consumption at the feeder level, which is the aggregate power consumption, and use this measurement to update the price. Therefore the algorithm can be easily adjusted to be the following algorithm without affecting any performance,

Algorithm 1: One-way Communication

- Initially, the utility company randomly chooses a price $p(0)$ and announce the price information to each user. Set $k = 0$.
- Upon receiving price $p(k)$ announced by the utility company, each customer i calculate and update its power consumption $q_i(k)$

$$q_i(k) = [(U_i')^{-1}(p(k))]^+, \tag{4}$$

- The utility company measure the total power consumption $q(k)$ at the substation feeder, which equals the total power consumption of users $\sum_i q_i(k)$ and updates the price according to

$$p(k+1) = [p(k) - \gamma(q(k) - Q)]^+, \tag{5}$$

  and announces price $p(k+1)$ to customers.
- let $k+1 \leftarrow k$ and repeat from step 2.

One problem of the implementation is the potential asynchronous updating of users, making the measurement of power consumption at the feeder $q(k)$ does not necessarily equal to the total power consumption of users $\sum_i q_i(k)$. That is, $q(k)$ might equal to $\sum_i q_i(k - \tau_i)$ where τ_i captures the asynchronous updating and even the time delay of the measurements. In the following, we will show that even there are asynchronous updating and time-delay, this one-way demand response scheme is still able to coordinate the users. (Lina says: need to change the way of presenting. just say it is hard to synchronize them)

Define $T_i \subset T$ as the set of time steps that user i updates and $T_u \subset T$ as the set of time steps that utility company measure the power consumption and updates the price. Mathematically, the following dynamics captures the asynchronous updating and time delay of the implementation.

Algorithm 2: One-way communication and asynchronous updating

At each time k, for the utility company,

$$p(k+1) = \begin{cases} [p(k) - \gamma(q(k-\tau) - Q)]^+, & \text{if } k \in T_u \\ p(k), & \text{else} \end{cases} \tag{6}$$

where τ denote the time delay between the time of measurement and updating the price. Here $q(k)$ denote the power measured at the feeder which is $\sum_i q_i(k)$. The dynamics of $q_i(k)$ is defined as follows,

For each user i,

$$q_i(k+1) = \begin{cases} [(U_i')^{-1}(p(k-\tau_i))]^+,, & \text{if } t \in T_i \\ q_i(k), & \text{else} \end{cases} \tag{7}$$

Here τ_i denote the time delay of passing price information from the feeder to user i.

Regardless of the asyncrhonous updating and time-delay, the DR will still converge to the optimal solution of problem (1) as long as the delay and time lag between any two consectutive updating is bounded.

Theorem 2 *The algorithm defined in (6) and (9) converges to optimal solution of (1) if the stepsize is small enough.*

This theorem demonstrates that Algorithm 1 is able to implemented using only one-way communication if we take advantages of the electricity network itself.

B. DR using one-way and single-bit communication

In the DR proposed in the preceding section, it requires only one-way communication where the utility company announcing the scale value of price information at each iteration. In this section, we target a more ambitious objective– only announcing a single-bit value at each step to coordinate the demand response.

Algorithm 3: Single bit communication

- Initially, all the users have a common initial price information $p_i(0) := p(0)$ and have a common step change Δp for the price. Note that a simple way to achieve this is to use a nominal price, e.g. $p = \$0.5$ per kwh, for each DR event. Set $k = 0$.
- Given the price $p_i(k)$, each customer i calculates and updates its power consumption $q_i(k)$

$$q_i(k) = [(U_i')^{-1}(p_i(k))]^+, \tag{8}$$

- The utility company measure the total power consumption $q(k)$ at the substation feeder, which equals the total power consumption of users $\sum_i q_i(k)$. If $q(k) < Q$, set the single bit number $s(k)$ be 0; if $q(k) > Q$, set the single bit number $s(k)$ be 1. Under both of the cases, the utility company send this single bit number $s(k)$ to all the customers. If $q(k) = Q$, the utility company does not send any more information to users.
- Upon receiving the information $s(k)$, each customer i update the price $p_i(k+1)$ according to:

$$p_i(k+1) = \begin{cases} p_i(k+1) + \Delta p, & \text{if } s(k) = 0 \\ p_i(k+1) - \Delta p, & \text{if} s(k = 1) \end{cases} \tag{9}$$

- let $k+1 \leftarrow k$ and repeat from step 2.

Through straightforward calculation, we know this DR algorithm with single bit communication is exactly the normalized sub-gradient descent algorithm [] for the dual optimization problem of (1). According to Theorem 2.1 in [1], we have the following theorem:

Theorem 3 *For any $\epsilon > 0$ and any $p^* \in M*$, there exist $k = k^*$ and $\bar{p}$ such that $D(\bar{p}) = D(p_{k^*})$ and $||\bar{p} - p^*|| \leq h(1+\epsilon)/2$.*

Moreover, if we use diminishing steplengths Δp_k instead of constant steplengths Δp, we have the following stronger result according to Theorem 2.2 in [1],

Theorem 4 *If $\lim_{k\to\infty} \Delta p_k = 0$ and $\sum_{k=0}^{\infty} \Delta p_k = +\infty$, then either an index $\bar{k}$ exists such that $x_{\bar{k}}$ is optimal or $lim_{k\to\infty} \min_{p\in M^*} ||p(k) - p|| = 0$ and $lim_{k\to\infty} D(p_k) = D^*$.*

V. Simulation

VI. Conclusion

References

[1] N. Z. Shor, *Minimization Methods for Non-Differentiable Functions*. Springer-Verlag New York, Inc., 1985.