

# Distributed Economic Model Predictive Control for Achieving Real-time Economic Dispatch with Frequency Control in Power Systems

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**Abstract**—The growing penetration of renewable and distributed energy resources introduces more and faster fluctuations into power systems. This poses a challenge for the existing system control to both satisfy system operation constraints and maintain an economic performance, especially during the transient operation. To overcome this, we consider Economic Model Predictive Control (EMPC), which is a dynamic optimization algorithm. To ease the computational burden of solving the optimization problem, the proposed controller is based on an inexact solution to an economic dynamic optimal control problem, which is obtained by reducing distributed dual iterations. This allows for a fully distributed implementation with only neighbor to neighbor communications. The theoretical properties of this controller include transient economic performance, transient constraint satisfaction and stability. The method also takes into account of many practical issues such as partial observations, model mismatches, and inexact predictions. We include a numerical study comparing this approach to a state of the art solution and show faster frequency stabilization and constraint satisfaction.

**Index Terms**—Predictive control, Distributed control, Distributed decision-making, Power generation dispatch, Power generation economics

## I. INTRODUCTION

The conventional frequency control for power networks consists of a hierarchical structure, in which different controllers operate at different time scales to achieve specific control goals [1], [2]. On the fastest level, primary frequency control absorbs load changes in a few seconds by adjusting the power. The frequency deviation is regulated on the second level by automatic generation control (AGC), which restores the nominal frequency within 5-10 minutes. The economically optimal operation point is computed in a centralized economic dispatch (ED), which operates at the slowest time scale. This computation is based on the Optimal Power Flow (OPF) problem, which can take minutes to hours to solve.

Due to the increasing proliferation of renewable and distributed energy generation, power networks are subject to fast and large power fluctuations. Since the economic performance

and the system constraints are only taken into consideration in the (slow) centralized ED, this can lead to a deterioration in the transient economic performance and to significant constraint violations. In order to solve this problem, a real time economic dispatch based on distributed computation is necessary. One option is to let secondary frequency control efficiently use information of the tertiary economic dispatch.

To address these problems, different control structures have been proposed in [3], [4], [5], [6], [7]. Most of these approaches pose a simplified economic dispatch as a *steady-state* optimization problem and implement the corresponding primal-dual algorithm as a real time controller for the system. If the uncontrolled system is linear and stable, the resulting closed-loop system was proven to converge to the optimal steady-state. Correspondingly, these controllers only provide guarantees for *steady-state* operation, which is almost never attained in reality due to time-varying fluctuations. Therefore, constraint satisfaction and economic efficiency in transient operation are of paramount importance, but cannot be guaranteed by the above mentioned algorithms.

A natural extension of these approaches is to consider model predictive control (MPC) [8], a dynamic optimization method, which provides theoretical guarantees during transient operation. While there have been previous MPC approaches for power networks [9], [10], they are designed to replace the AGC and achieve better stability properties. They are typically not designed to achieve real time economic dispatch.

*Contribution of this paper.* We propose a distributed economic model predictive control (DEMPC) [11] based on inexact dual minimization to approximately solve the economic dispatch problem in real time. Instead of driving the system to some predetermined setpoint, we directly consider an economic cost which can be specified by the system operator. By minimizing this cost function the economic MPC ensures bounds on the transient economic performance and implicitly stabilizes the system to the associated optimal steady-state. One of the core challenges to apply MPC to fast and large systems is the computation burden of solving the optimization in real time. We use dual distributed optimization algorithms to enable the application to large scale networks. Furthermore, we adjust the MPC optimization problem to explicitly compensate inexact solutions in the distributed dual optimization due to real time requirements. Thus, the resulting DEMPC can be implemented in a distributed fashion with limited neighbor to neighbor communication. Furthermore, the controller guarantees con-

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straint satisfaction, stability and economic performance during transient operation, which are desired properties to handle fast and large fluctuations. Compared to existing MPC approaches for power networks, the presented approach is scalable, explicitly takes economic performance into consideration and ensures constraint satisfaction under inexact minimization. We demonstrate the effectiveness of the proposed DEMPC approach using nonlinear simulations with the IEEE 39 bus system. Our method also takes into account of many practical issues such as partial observations, model mismatches, inexact predictions, etc.

Lastly, we discuss the difference between this paper and the conference paper [12]. Though [12] also studied EMPC, it only focused on linear dynamical systems and the analysis were only about ideal cases without much details. More importantly, it neglected many important issues for real-world implementation. In contrast, this paper uses the nonlinear dynamical model, articulates the method in a much more rigorous and organized way, and addresses many important practical issues in a principled way including partial observations, model mismatches, prediction errors, etc. Note that these practical issues require us to redesign man parts of EMPC and re-derive the performance guarantees. Correspondingly, all the simulations and discussions are new. As shown, this paper is substantially different from [12], both in technical contributions and in writing.

The remainder of this paper is structured as follows: Section II presents the economic dispatch problem and discusses some preliminaries. Section III presents the inexact DEMPC with the corresponding distributed dual iteration. Section IV compares the proposed controller to a state of the art controller in simulation scenarios. Section V concludes the paper.

### A. Notation

The real numbers are denoted by  $\mathbb{R}$ , the natural numbers by  $\mathbb{N}$  and the identity matrix by  $I_n \in \mathbb{R}^{n \times n}$ . The component wise absolute value of a vector  $x \in \mathbb{R}^n$  or matrix  $x \in \mathbb{R}^{n \times m}$  is denoted by  $|x|$ . A power network can be modeled as an undirected graph  $(\mathcal{N}, \mathcal{E})$ , where each node  $i \in \mathcal{N}$  represents a bus and the edges  $(i, j) \in \mathcal{E} \subset \mathcal{N} \times \mathcal{N}$  represent the physical couplings with the line power flow  $P_{ij}$ . The buses are separated into generators  $\mathcal{G}$  and loads  $\mathcal{L}$  with  $\mathcal{N} = \mathcal{G} \cup \mathcal{L}$ . The neighbors of bus  $i$  are given by  $\overline{\mathcal{N}}_i = \{j \in \mathcal{N} | (i, j) \in \mathcal{E}\}$  and the full neighborhood including bus  $i$  is given by  $\mathcal{N}_i = \overline{\mathcal{N}}_i \cup \{i\}$ . The overall state  $x \in \mathbb{R}^n$  of the power network consists of local states  $x_i \in \mathbb{R}^{n_i}$  of bus  $i$  with  $x = [x_1^\top, \dots, x_{|\mathcal{N}|}^\top]^\top$ ,  $n = \sum_{i \in \mathcal{N}} n_i$ . The states of the full neighborhood  $\mathcal{N}_i$  are denoted by  $x_{\mathcal{N}_i} \in \mathbb{R}^{n_{\mathcal{N}_i}}$  with  $n_{\mathcal{N}_i} = \sum_{j \in \mathcal{N}_i} n_j$ . We introduce projection matrices  $W_i \in \{0, 1\}^{n_{\mathcal{N}_i} \times n}$ , such that  $x_{\mathcal{N}_i} = W_i x$ .

## II. PRELIMINARIES AND CONTROL OBJECTIVES

We study generation control when there are changes in net loads, e.g, load minus renewable generation, from their nominal (operating) points.<sup>1</sup> To simplify the notation, all of the

<sup>1</sup>Our MPC approach and the results derived in this paper can be easily extended to load control. We focus on the generation control for the ease of exposition.

variables in this paper represent deviations from their nominal (operating) values. In practice those nominal values are usually determined by the last economic dispatch problem [2].

In the following, we provide preliminaries and our control objectives, which are necessary for a successful implementation of the model predictive controller (MPC), which is described in Section III.

### A. Dynamical Model for Prediction

One important factor for successfully implementing MPC is an accurate dynamical model to simulate the system dynamics. This model is used to predict the system state under designed control inputs. In the following, we derive a linear distributed discrete time prediction model for the nonlinear continuous time power dynamics.

1) *Nonlinear network model:* The generator dynamics  $i \in \mathcal{G}$  can be described with the following network model [6], [7]

$$\dot{P}_i^M = -\frac{1}{T_i}(P_i^M - P_i^C), \quad (1a)$$

$$\dot{\omega}_i = -\frac{1}{M_i} \left( D_i \omega_i + P_i^L - P_i^M + \sum_{(i,j) \in \mathcal{E}} P_{ij} \right), \quad (1b)$$

$$\dot{\delta}_i = \omega_i, \quad (1c)$$

with the mechanical power  $P_i^M$ , the frequency deviation  $\omega_i$ , the phase shift  $\delta_i$ , the power change control command  $P_i^C$ , the power load  $P_i^L$  and the line power flow  $P_{ij}$ . The bus parameters are the generator inertia  $M_i$ , the damping constant  $D_i$  and the time constant  $T_i$ . Note that the generator turbine-governor model is simplified to the first order model in equation (1). The line power flow is given by the following nonlinear equation in dependence of the phase shift  $\delta_{ij} = \delta_i - \delta_j$ :

$$P_{ij} = \frac{V_i V_j}{l_{ij}^2 + r_{ij}^2} (l_{ij} \sin(\delta_{ij}) - r_{ij} \cos(\delta_{ij})) - P_{ij}^0, \quad (i, j) \in \mathcal{E}. \quad (2)$$

The corresponding line parameters are the bus voltage  $V_i$  and  $V_j$ , the line inductance  $l_{ij}$ , the line resistance  $r_{ij}$  and the nominal line power flow  $P_{ij}^0$  corresponding to the nominal phase shift  $\delta_{ij}^0$ .

With the local state  $x_i = [P_i^M, \omega_i, \delta_i] \in \mathbb{R}^3$ , the control input  $u_i = P_i^C \in \mathbb{R}$  and the external disturbance  $d_i = P_i^L \in \mathbb{R}$ , we can write the generator dynamics in the following form

$$\dot{x}_i = f_{c,i}(x_{\mathcal{N}_i}, u_i, d_i), \quad i \in \mathcal{G}.$$

The load dynamics  $i \in \mathcal{L}$  are described by the power balance equation

$$\dot{\delta}_i = \omega_i = -\frac{1}{D_i} \left( P_i^L + \sum_{(i,j) \in \mathcal{E}} P_{ij} \right).$$

Correspondingly, the local state  $x_i = \delta_i \in \mathbb{R}$  describes the load dynamics with the coupled model:

$$\dot{x}_i = f_{c,i}(x_{\mathcal{N}_i}, d_i), \quad i \in \mathcal{L}.$$

The overall system is described by

$$\dot{x} = f_c(x, u, d).$$

2) *Successive linearization*: This nonlinear model tends to be too complex for the controller design. In order to avoid a large model mismatch, the line power flow is successively linearized around the current phase

$$\Delta P_{ij} \approx \underbrace{\frac{V_i V_j}{l_{ij}^2 + r_{ij}^2} (l_{ij} \cos(\delta_{ij}) + r_{ij} \sin(\delta_{ij}))}_{=B_{ij}(\delta_{ij})} \Delta \delta_{ij}.$$

For small to medium size phase shifts  $\Delta \delta_{ij}$  we have  $B_{ij}(\delta_{ij}) \approx B_{ij}(\delta_{ij}^0)$  and thus the line power flow is affine in the phase shift  $\Delta \delta_{ij}$ :

$$P_{ij} \approx \tilde{P}_{ij} + B_{ij} \Delta \delta_{ij},$$

where  $\tilde{P}_{ij}$  denotes the current line power flow.

With this, we have an affine distributed model:

$$\dot{x} = A_c x + B_c u + G_c d + c_c.$$

The affine term  $c_c$  depends on  $\tilde{P}_{ij}$  and is updated online based on the measurement.

3) *Discrete time model*: In order to enable efficient predictions, we use a distributed discrete time model of the power network. Typical methods to discretize distributed system dynamics are explicit discretization schemes, like the Euler method. These methods keep the structure in the dynamics and for a small enough step size  $h$  also approximate the continuous dynamics sufficiently well.

The load dynamics are very fast ( $\tau_i \leq 0.1ms$ ) and are thus often approximated as a differential algebraic equation (DAE) [13], with settled load dynamics  $\omega_i = 0, i \in \mathcal{L}$ . Standard explicit discretization schemes are unsuitable for such stiff power networks.

The problem of stiff dynamics in power networks is well known. To overcome this issue we use the implicit trapezoidal method [14], [15], which is often used for the simulation of power networks. The system is approximated by the linear equality constraint

$$\underbrace{\left( I_n - \frac{h}{2} A_c \right)}_F x^+ = \underbrace{\left( I_n + \frac{h}{2} A_c \right)}_A x + \underbrace{h B_c}_B u + \underbrace{h G_c}_G d + \underbrace{h c_c}_c, \quad (3)$$

with the step size  $h$ . Due to the distributed structure of  $A_c$ ,  $B_c$ ,  $G_c$  the dynamic equality constraint (3) has the following distributed structure

$$F_{\mathcal{N}_i} x_{\mathcal{N}_i}^+ = A_{\mathcal{N}_i} x_{\mathcal{N}_i} + B_i u_i + G_i d_i + c_i. \quad (4)$$

The following controller design is based on the linear discrete-time model (4), while the true system is assumed to be described by the nonlinear continuous-time model (1), (2). In Section III-D we discuss how we can systematically account for this model mismatch in the controller design.

4) *Integration of existing primary frequency control*: Typically, there exist primary frequency controllers  $u_i = K_{\mathcal{N}_i} x_{\mathcal{N}_i}$ , which we would like to incorporate into the MPC design. Some recent publications on the design of primary frequency controllers are given in [16], [17], [18]. We can use such an

existing feedback by implementing the control input  $u_i(t) = K_{\mathcal{N}_i} x_{\mathcal{N}_i}(t) + v_i(t)$ , where  $v_i(t)$  is the new control input which is calculated based on the MPC. The model (4) correspondingly changes to the pre-stabilized dynamics including the feedback. This way, the MPC is put on top of the existing control structure and can use existing schemes. Thus, the predictive controller is only necessary to ensure constraint satisfaction and improve the economic<sup>2</sup> performance. In addition, the incorporation of an existing feedback  $K$  can significantly reduce the constraint tightening in Section III and might allow for an implementation of the MPC with a longer sampling time (compare Section III-E3).

## B. Estimation

We have a simple model (4) to predict the future response of the power network for a given input trajectory<sup>3</sup>  $u(\cdot|t)$ . However, this requires the current state  $x(t)$  and the power load  $d(t)$  to be known. In practice, only the line power flow  $P_{ij}$  and the frequency  $\omega_i$  can be measured. Thus, an augmented distributed observer needs to be implemented to compute the state estimate  $\hat{x}(t)$  and load estimate  $\hat{d}(t)$  based on the measured variables. In [19, Sec. 5.4.2] a distributed augmented Lunenberg observer is designed with distributed linear matrix inequalities (LMIs) to estimate the state  $x(t)$  and a randomly changing load  $d(t)$  for a power network. With such an observer, we have real time estimates of the state and load with bounded errors despite model mismatch and noise. There exist a variety of different, potentially better, distributed observers that can be designed, which is however not the focus of this paper.

In order to predict the future output response, we also need to know the future power load  $d(\cdot|t)$ . For the purpose of performing theoretical analysis, we assume that the load  $d$  stays constant over the prediction horizon. Small deviations can be treated as a model mismatch. In case of a large load change, there might exist some predictive knowledge which can be directly incorporated in the prediction of the system response and thus in the MPC scheme.

## C. Real time economic dispatch

In the following, we formulate our objectives of the real time economic dispatch in power networks. The control specifications can be separated into three goals: constraint satisfaction; economic performance; and frequency regulation.

1) *Constraints*: The first goal is to ensure constraint satisfaction during transient operation. We consider the following hyper-box constraints for the state and input

$$P_i^C \in [P_{i,\min}^C, P_{i,\max}^C], \quad P_i^M \in [P_{i,\min}^M, P_{i,\max}^M], \quad (5)$$

$$\omega_i \in [\omega_{i,\min}, \omega_{i,\max}], \quad P_{ij} \in [P_{ij,\min}, P_{ij,\max}].$$

The considered constraints for the power generation  $P_i$  and line power flow  $P_{ij}$  are standard for the economic dispatch [6],

<sup>2</sup>If possible, one should use a stabilizing feedback  $K$ , such that the result in Lemma 1 still holds. This is for example the case if the controller only depends on frequency deviations, like droop control.

<sup>3</sup>We denote predicted trajectories by  $u(\cdot|t) = \{u(0|t), \dots, u(N-1|t)\} \in \mathbb{R}^{m \times N}$ , where  $u(k|t)$  is the predicted input at time  $t+k$ .

[7]. The constraint on the frequency deviation, on the other hand, is only relevant for transient operation, but can still be of high practical relevance.

Constraint violations in the line power flow or the frequency deviation, even for a short period of time, might cause a power outage. Correspondingly, it is crucial that the posed constraints are satisfied at all times during transient operation.

2) *Economic performance*: The desired behavior of the power network needs to be captured in a suitable distributed economic cost  $l(x, u)$ . In particular, this economic cost should be chosen such that by minimizing the transient economic cost, the MPC scheme implicitly stabilizes the optimal setpoint associated with the economic dispatch.

Each generator  $i \in \mathcal{G}$  incurs a certain cost  $\alpha_i(P_i^M)$  when the power generation is  $P_i^M$ . This cost is typically specified by a user. Different interpretations and the relation to the economic dispatch can be found in [6]. This cost function is typically assumed to be continuous, differentiable and (strictly) convex.

In addition, we consider that the power command  $P_i^C$  incurs the same cost with some weighting factor  $\gamma > 0$ , i.e.  $\gamma\alpha_i(P_i^C)$ . This can be thought of as the cost associated purely with the power command  $P_i^C$  and thus with changing the current level of power generation  $P_i^M$ . Alternatively, this can be interpreted as an additional (optional) tuning variable that allows a user to tune the transient response of the controller by penalizing large control inputs, which correspond to fast changes in the power generation.

Furthermore, large frequency deviations  $\omega_i$  are undesirable and should be suppressed. Thus, the economic cost includes a quadratic cost  $\beta\omega_i^2$  with some user specified parameter  $\beta > 0$ . With this, the economic cost is given by

$$l(x, u) = \sum_{i \in \mathcal{G}} l_i(x_i, u_i) = \sum_{i \in \mathcal{G}} \alpha_i(P_i^M) + \gamma\alpha_i(P_i^C) + \beta\omega_i^2. \quad (6)$$

This economic cost has a convex, distributed structure, which makes it suitable for distributed (dual) optimization algorithms. It is important to note, that the MPC minimizes the predicted economic cost  $l(x(\cdot|t), u(\cdot|t))$  over some transient prediction horizon, instead of computing and directly stabilizing the optimal steady-state.

3) *Frequency regulation*: The frequency constraint (5) ensures bounds on the frequency deviation and the economic cost (6) incentivizes small frequency deviation. Nevertheless, for constant shifts in the power load, the economically optimal behavior is to stabilize a common, not necessarily zero, frequency deviation. In [7] a similar cost is considered and the minimizer is shown to be a frequency synchronized solution, similar to droop control. To ensure that no such offset in the frequency deviation persists, we add the following constraint

$$\delta_i \in [\delta_{\min}, \delta_{\max}], \quad (7)$$

which implicitly introduces a transient constraint on the frequency deviation and ensures that the system is driven to steady-state operation in the absence of dynamic load changes. **This constraint can be thought of as the MPC based analog to AGC, which yields an integral like control action, compare simulations in Section IV.**

The constraints (5), (7) can be modeled with the following coupled polytopic state constraint sets

$$\mathcal{X}_{\mathcal{N}_i} = \{x_{\mathcal{N}_i} | H_{\mathcal{N}_i} x_{\mathcal{N}_i} \leq h_i\}, \quad i \in \mathcal{N}, \quad h_i \in \mathbb{R}^{p_i}.$$

We also introduce the overall state constraint set  $\mathcal{X}$ :

$$\mathcal{X} = \{x | W_i x = x_{\mathcal{N}_i} \in \mathcal{X}_{\mathcal{N}_i}, \forall i \in \mathcal{N}\}.$$

The input constraint sets are defined similarly as

$$\mathcal{U}_i = \{u_i | L_i u_i \leq l_i\}, \quad l_i \in \mathbb{R}^{q_i}, \quad \mathcal{U} = \mathcal{U}_1 \times \dots \times \mathcal{U}_{|\mathcal{N}|}.$$

Note that we require satisfaction of the constraints at all time instances, i.e.  $(x(t), u(t)) \in \mathcal{X} \times \mathcal{U}, \forall t \geq 0$ , not just at stationary operation.

Given the economic cost (6), the constraint sets and a given power load  $d$ , we can formally define the optimal steady-state

$$\begin{aligned} (x^e, u^e) &= \arg \min l(x, u) \\ \text{s.t. } (x, u) &\in \mathcal{X} \times \mathcal{U}, \\ Fx &= Ax + Bu + Gd + c. \end{aligned} \quad (8)$$

**Lemma 1.** *For a constant load  $d$ , the optimal steady-state  $(x^e, u^e)$  in (8) is independent of the tuning parameters  $\beta, \gamma$  and is equivalent to the solution of the economic dispatch problem [6, Eq. (5)] with the cost function  $\alpha_i(P_i^M)$ .*

*Proof.* First note, that  $\dot{\delta}_i = \omega_i$  implies that steady-state frequency deviation is zero. Furthermore, the stationary input satisfies  $P_i^C = P_i^M$ . Thus, the steady-state cost is given by  $l(x^e, u^e) = \sum_{i \in \mathcal{G}} (1 + \gamma)\alpha_i(P_i^M)$  and the minimizer is independent of  $\beta, \gamma$ .  $\square$

The tuning parameters  $\beta, \gamma$  do not affect the optimal steady-state, hence they can be freely tuned to shape the transient response of the controller. High values of  $\gamma$  lead to a smooth but slow response. Large values of  $\beta$  lead to small frequency deviations but also slower convergence to the optimal steady-state. As we will see later, the MPC scheme implicitly "finds" the optimal steady-state without explicitly computing it. Thus, both transient and stationary control goals can be treated by separate tuning factors with a unified control approach.

### III. INEXACT DISTRIBUTED ECONOMIC MPC

Given the preliminaries in Section II, we can formulate the DEMPC. First, an idealized MPC scheme with corresponding theoretical properties is discussed. Thereafter, the mathematical formulation for the proposed inexact DEMPC scheme is presented. Then, the distributed dual iteration with a corresponding stopping condition is described and the computation of the constraint tightening is explained. Finally, the offline procedure and the closed-loop operation are summarized and the theoretical properties are discussed.

#### A. Idealized model predictive control formulation

Before presenting the proposed inexact DEMPC scheme, we briefly discuss the properties of an idealized MPC scheme. By idealized, we mean that there exists no model mismatch and that the optimization problem is solved exactly, which are standing assumptions in much of the MPC literature.

Given a receding horizon window  $N$ , model predictive control (MPC) is characterized by solving an optimization problem at each time step  $t$  and then applying the first part of the computed optimal input trajectory to the system. The corresponding standard optimization problem is given by

$$\min_{u(\cdot|t), x(\cdot|t)} \sum_{k=0}^{N-1} \sum_{i \in \mathcal{G}} l_i(x_i(k|t), u_i(k|t)) \quad (9a)$$

$$\text{s.t. } x(0|t) = x(t), \quad (9b)$$

$$Fx(k+1|t) = Ax(k|t) + Bu(k|t) + Gd(k|t) + c(t), \quad (9c)$$

$$P_i^M(k|t) \in [P_{i,\min}, P_{i,\max}], \omega_i(k|t) \in [\omega_{i,\min}, \omega_{i,\max}], \quad (9d)$$

$$P_i^C(k|t) \in [P_{i,\min}^C, P_{i,\max}^C], P_{ij}(k|t) \in [P_{ij}^{\min}, P_{ij}^{\max}], \quad (9e)$$

$$k = 0, \dots, N-1, \quad i \in \mathcal{N}, \quad j \in \mathcal{N}_i,$$

with the current estimated state  $x(t)$ . **The constraints (9d)–(9e) can be written compactly as  $x(k|t) \in \mathcal{X}$ ,  $u(k|t) \in \mathcal{U}$ .**

The solution to this optimization problem are optimal state and input trajectories  $x^*(\cdot|t)$ ,  $u^*(\cdot|t)$  that satisfy these constraints and minimize the economic cost  $l(x, u)$  over the prediction horizon  $N \in \mathbb{N}$ . **Since system security is formulated in terms of hard constraints (9d)–(9e), the controller always prioritizes safety over economic performance.** In a standard MPC setup the optimization problem (9) is solved exactly at each time step  $t$  and the **first step of the optimal open-loop input sequence  $u^*(\cdot|t)$  is applied to the system, i.e.,  $u(t) = u^*(0|t)$ .** **Crucially, MPC is a receding horizon strategy and the constant prediction horizon  $N$  is continuously shifted to start at the current time  $t$  in closed-loop operation. In Section III-E3 we discuss how this can be generalized to solving the optimization problem only every  $M$  time steps and applying the first  $M$  steps of the optimized input sequence.**

In the following, we assume that  $\alpha_i$  is a strictly convex quadratic<sup>4</sup> function. The following proposition summarizes the properties of the closed-loop MPC under idealized conditions.

**Proposition 2.** [21], [22] [23, Thm. 5] *Consider a constant load  $d(t)$ . Suppose that the unknown optimal steady-state (8)  $x^e$  lies strictly in the constraint set and that  $(F, A, B)$  is stabilizable. Assume further, that the prediction model is identical to the real system and the optimization problem (9) is solved exactly in each time step  $t$ . Then, for any set of feasible initial conditions, there exists a sufficiently large  $N$ , such that the MPC problem (9) is recursively feasible for the closed-loop system and the constraints are satisfied for all  $t \geq 0$ . Furthermore, the optimal steady-state  $x^e$  is practically asymptotically stable under the closed-loop dynamics.*

By recursive feasibility, we mean the property that the optimization problem remains feasible, i.e. there exists a solution to the optimization problem at each time step  $t$ . Practical asymptotic stability, as for example defined in [21, Def. 2.2], implies that the closed-loop system stabilizes a ball centered around the optimal steady-state. In this case, the size of this ball depends on the prediction horizon  $N$ . The larger

<sup>4</sup>If the nonlinear cost  $l(x, u)$  is strictly convex (but not necessarily quadratic), the theoretical properties remain valid. A strictly convex quadratic economic cost can be interpreted as a tracking cost of a desired but unreachable setpoint [20].

the prediction horizon  $N$ , the closer we get to the optimal steady-state. From a practical point of view, the difference between practical asymptotic stability and asymptotic stability may be negligible since the system is constantly subject to changes and fluctuations.

## B. Inexact model predictive control formulation

From a practical point of view, assuming no model mismatch and exact optimization in each time step is unrealistic. Thus, we consider a modified formulation based on the theoretical results<sup>5</sup> in [24]. This inexact DEMPC scheme is characterized by approximately solving the following distributed optimization problem at each time step  $t$

$$\min_{u(\cdot|t), x(\cdot|t)} \sum_{k=0}^{N-1} \sum_{i \in \mathcal{G}} l_i(x_i(k|t), u_i(k|t)) \quad (10)$$

$$\text{s.t. } x(0|t) = x(t),$$

$$|Ax(k|t) + Bu(k|t) + Gd(k|t) + c(t) - Fx(k+1|t)| \leq w_k,$$

$$w_k = (N-1-k) \cdot v, \quad k = 0, \dots, N-1,$$

$$x(k|t) \in \bar{\mathcal{X}}_k, \quad u(k|t) \in \mathcal{U}, \quad k = 0, \dots, N-1.$$

Compared to the standard formulation (9), the dynamic equality constraint (3) is relaxed by some positive vector  $v$ ,  $w_k \in \mathbb{R}^n$ . Furthermore, the state constraints are replaced by tightened constraint sets  $\bar{\mathcal{X}}_k$ . These two modifications are needed to ensure recursive feasibility and constraint satisfaction for the closed-loop system based on the MPC with inexact minimization and are discussed in more detail in the subsequent subsections (compare [24]). In the following, we only consider the proposed problem (10), since the standard problem (9) does not ensure theoretical properties of the closed-loop system under realistic conditions, such as model mismatch and inexact optimization.

## C. Distributed dual optimization

To enable the practical application of the proposed inexact DEMPC, an approximate solution to the distributed quadratic program (QP) (10) needs to be computed in real time in a scalable distributed fashion. Dual optimization algorithms enable a fully distributed solution based on dual decomposition, only require neighbor to neighbor communication and are thus suitable for large power networks. There exists a variety of dual decomposition methods, such as dual gradient [26], proximal decomposition [27] and alternating minimization [28], [29]. **Such dual decomposition methods have also been considered in many distributed MPC implementations, compare [27], [28], [30], [31].** The alternating direction method of multipliers (ADMM) [29] is a particular dual optimization method, which can often achieve reasonable results with few iterations and is thus considered here.

A corresponding formulation of distributed MPC with ADMM has been provided in [30] and is adopted here. We define a shared variable  $z = \{x(\cdot|t), u(\cdot|t)\}$  and local

<sup>5</sup>In [24] the issue of constraint satisfaction and recursive feasibility under inexact dual minimization is considered. Within this framework, additional model mismatch can be accounted for quite naturally, similar to [25].

variables  $y_i = \{x_{\mathcal{N}_i}^i(\cdot|t), u_i(\cdot|t)\}$ ,  $i \in \mathcal{N}$ . The local variables contain the local input trajectory  $u_i(\cdot|t)$  and neighboring<sup>6</sup> state trajectories  $x_{\mathcal{N}_i}^i(\cdot|t)$ , as optimized by bus  $i$ . To ensure that we recover the original solution, a consistency constraint  $x_i^i(\cdot|t) = x_i^j(\cdot|t)$  is used, which can be written as

$$E_i z = y_i, \quad i \in \mathcal{N}. \quad (11)$$

The MPC optimization problem (10) is equivalent to

$$\begin{aligned} \min_{y, z} \sum_{i \in \mathcal{N}} J_i(y_i) &= \sum_{i \in \mathcal{N}} \sum_{k=0}^{N-1} l_i(x_i^i(k|t), u_i(k|t)) \\ \text{st. } y_i &\in \mathcal{Y}_i(x_i(t)), \quad y_i = E_i z, \quad i \in \mathcal{N}, \end{aligned} \quad (12)$$

with the local polytopic constraint set  $\mathcal{Y}_i(x_i)$ :

$$\begin{aligned} \mathcal{Y}_i(x_i) &= \{y_i | x_i(0|t) = x_i, x_{\mathcal{N}_i}^i(k|t) \in \bar{\mathcal{X}}_{\mathcal{N}_i, k}, \\ &|A_{\mathcal{N}_i} x_{\mathcal{N}_i}^i(k|t) + B_i u_i(k|t) + G_i d_i(k|t) + c_i(t) \\ &- F_{\mathcal{N}_i} x_{\mathcal{N}_i}^i(k+1|t)| \leq w_{i, k}, \quad w_{i, k} = (N-1-k) \cdot v_i, \\ &u_i(k|t) \in \mathcal{U}_i, \quad k = 0, \dots, N-1\}. \end{aligned} \quad (13)$$

Here,  $v_i$ ,  $w_{i, k} \in \mathbb{R}^{n_i}$  denote the local variables of  $v$ ,  $w_k$  in (10). The local augmented Lagrangian  $\mathcal{L}_{i, \rho}$  is given by

$$\mathcal{L}_{i, \rho}(y_i, z, \lambda_i) = J_i(y_i) + \lambda_i^\top (y_i - E_i z) + \frac{\rho}{2} \|E_i z - y_i\|_2^2,$$

with a tuning parameter  $\rho > 0$  and the local dual variable  $\lambda_i$ . The distributed ADMM iteration to solve (12) is summarized by the following algorithm:

---

**Algorithm 1** Alternating Direction Method of Multipliers

---

**Initialization:** set  $\rho > 0$ , set initial guess  $z^0$ ,  $\lambda_i^0$ .

- 1: **for**  $p = 0, \dots$  until stopping condition (14) is fulfilled **do**
  - 2:  $y_i^{p+1} = \arg \min_{y_i \in \mathcal{Y}_i(x_i)} \mathcal{L}_{i, \rho}(y_i, z^p, \lambda_i^p)$
  - 3: communicate  $y_i^{p+1}$  to neighbors  $j \in \mathcal{N}_i$
  - 4:  $z_i^{p+1} = \frac{1}{|\mathcal{N}_i|} \sum_{j \in \mathcal{N}_i} E_{ji}^\top (y_j^{p+1} + \frac{1}{\rho} \lambda_{ji}^p)$
  - 5: communicate  $z_i^{p+1}$  to neighbors:  $j \in \mathcal{N}_i$
  - 6:  $\lambda_i^{p+1} = \lambda_i^p + \rho (y_i^{p+1} - E_i z^{p+1})$
  - 7: **end for**
- 

The ADMM iteration asymptotically converges to the optimal solution of the original optimization problem [29]. Due to real time requirements, we stop the ADMM optimization once the following stopping condition is satisfied

$$\|E_i z - y_i\|_\infty \leq \epsilon, \quad \epsilon > 0, \quad (14)$$

which can be checked efficiently in a distributed fashion. The resulting inexact state and input trajectory are denoted by  $u_\epsilon(\cdot|t)$  and  $x_\epsilon(\cdot|t)$ , with  $x_{i, \epsilon}(k|t) = x_i^i(k|t)$ . **Crucially, without additional modifications, the application of the resulting (inexact) input trajectory  $u_\epsilon$  does not ensure constraint satisfaction and is thus not safe.**

<sup>6</sup>Using the fact that only the neighboring phase  $\delta_j$  is relevant for both the dynamic coupling and the coupled state constraints, it suffices to include the neighboring phase  $\delta_j$  instead of the full neighboring states  $x_j$ . **Furthermore, only the relative angle  $\delta_{ij}$  (instead of the absolute value  $\delta_j$ ) is relevant.**

#### D. Constraint tightening

Due to both, inexact minimization and model mismatch, the predicted trajectories differ from the resulting system trajectories. Thus, to ensure that the resulting system trajectories satisfy the constraints we have to consider tightened constraints for the predicted trajectories.

1) *Inexact solution:* Due to the inexact solution (14), the consistency constraints (11) are not satisfied and the inexact trajectory satisfies the following perturbed dynamic equation:

$$\begin{aligned} |A_{\mathcal{N}_i} x_{\mathcal{N}_i, \epsilon}(k|t) + B_i u_{i, \epsilon}(k|t) + c_i(t) \\ + G_i d_i(k|t) - F_{\mathcal{N}_i} x_{\mathcal{N}_i, \epsilon}(k+1|t)| \leq w_{i, k} + v_i = w_{i, k-1}, \end{aligned} \quad (15)$$

with the bounded error

$$v_i := 2\epsilon \left( \sum_{j \in \mathcal{N}_i} |A_{ij}| \mathbf{1}_{n_j} + \sum_{j \in \mathcal{N}_i} |F_{ij}| \mathbf{1}_{n_j} \right) \in \mathbb{R}^{n_i}. \quad (16)$$

Compared to the posed dynamic constraint, the early termination (14) implies an additional constraint violation of  $v_i$ . The relaxation of the dynamic constraint in (10) in combination with the inexact optimization (14) implies the following error in the inexact minimization

$$|Ax(t) + Bu(t) + Gd(t) + c(t) - Fx_\epsilon(1|t)| \leq w_0 + v.$$

This error can be represented with the hyperbox

$$\mathcal{W}_{\text{inexact}} = \{w \in \mathbb{R}^n | |w| \leq w_0 + v = N \cdot v\}.$$

2) *Model mismatch:* In addition to the error caused by the inexact solution, there typically exists some model mismatch. By that, we mean that the discrete-time prediction model differs from the true physical system (due to nonlinearities, discretization and unmodelled dynamics). We assume that this error in the model can be bounded with some polytopic set  $\mathcal{W}_{\text{model}}$ , such that the real system satisfies

$$Fx(t+1) = Ax(t) + Bu(t) + Gd(t) + c(t) + w_{\text{model}}(t),$$

with some  $w_{\text{model}}(t) \in \mathcal{W}_{\text{model}}$ . **For practical purposes, we can use simulations experiments to obtain simple estimates for this model mismatch.**

3) *Combined prediction error:* By combining the model mismatch  $w_{\text{model}}$  with the error caused by the inexact minimization  $w_{\text{inexact}}$ , we can define the true prediction error

$$\mathcal{W}_{\text{prediction}} = \mathcal{W}_{\text{model}} \oplus \mathcal{W}_{\text{inexact}}.$$

The closed-loop system based on the inexact MPC input  $u(t) = u_\epsilon(0|t)$  satisfies

$$Fx(t+1) = Fx_\epsilon(1|t) + w_{\text{prediction}}, \quad w_{\text{prediction}} \in \mathcal{W}_{\text{prediction}}.$$

To ensure that the real system trajectory satisfies the state constraints despite this error in the predictions, the constraints are tightened using the  $k$ -step support function [32]. For some

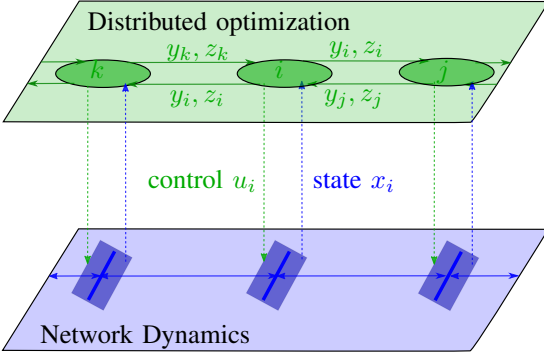


Fig. 1. Information flow: The local bus  $i$  sends the estimate of the local state  $x_i$  to a local computational unit. The computational units communicate with neighboring buses and exchange predicted trajectories  $y_i, z_i$  needed for the distributed optimization. The computation yields a local control input  $u_i$ , which is applied at bus  $i$ .

$a \in \mathbb{R}^n$  and  $k \in \mathbb{N}$ , the  $k$ -step support function is defined as

$$\begin{aligned} \sigma(a, k) &= \sup_{w \in \mathcal{W}_{\text{prediction}}^k} a^\top y(k), \\ \text{st. } y(0) &= 0, \\ Fy(l+1) &= Ay(l) + w(l), \quad l = 1, \dots, k-1. \end{aligned}$$

The evaluation of the  $k$ -step support function amounts to solving a distributed linear program (LP) offline. The local tightened constraints  $\bar{\mathcal{X}}_{\mathcal{N}_i, k}$  for the prediction step  $k$

$$\bar{\mathcal{X}}_{\mathcal{N}_i, k} = \{x_{\mathcal{N}_i} | H_{\mathcal{N}_i, j} x_{\mathcal{N}_i} \leq \bar{h}_{i, j, k}, j = 1, \dots, p_i\},$$

are computed according to

$$\bar{h}_{i, j, k} = h_{i, j} - \sigma((H_{\mathcal{N}_i, j} W_i)^\top, k), \quad j = 1, \dots, p_i, \quad i \in \mathcal{N}. \quad (17)$$

### E. Closed-loop operation

The initialization and closed-loop operation of the MPC scheme are summarized in the following two algorithms. **The corresponding information flow is sketched in Figure 1.**

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#### Algorithm 2 Offline distributed synthesis

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- 1: Set up Model with step size  $h$  (Sec. II-A).
  - 2: Design observer (Sec. II-B).
  - 3: Estimate model mismatch  $\mathcal{W}_{\text{model}}$ .
  - 4: Set accuracy  $\epsilon$ , prediction horizon  $N$ , penalty  $\rho$ .
  - 5: Compute dynamic inaccuracy  $v_i$  (16),  $\mathcal{W}_{\text{inexact}}$ .
  - 6: Compute tightened constraints (17).
- 

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#### Algorithm 3 Online DEMPC, execute at every time step $t$

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- 1: Estimate state  $x(t)$  and load  $d(\cdot|t)$ . Update  $c(t)$  (3).
  - 2: Compute candidate solution  $z^0, \lambda_i^0$ .
  - 3: Approximately solve Problem (10) with Alg. 1.
  - 4: Apply control input:  $u_i(t) = u_{i, \epsilon}(0|t)$ .
- 

1) *Dual optimization: Initialization and parameter tuning:* The number of necessary iterations in Algorithm 1 depends strongly on the initialization  $\lambda_i^0, z^0$  and the penalty factor  $\rho$ .

A simple<sup>7</sup> initialization of  $z^0$  is given by the previous optimal solution  $x^*(\cdot|t), u^*(\cdot|t)$  shifted along the prediction horizon, i.e.

$$\begin{aligned} u(k|t+1) &= u^*(k+1|t), \quad u(N-1|t+1) = u(N-1|t), \\ x(k|t+1) &= x^*(k+1|t), \quad k = 0, \dots, N-2. \end{aligned}$$

The dual variables  $\lambda_i^0$  can be similarly shifted and appended by 0. In [33], the issue of selecting  $\rho$  and pre-conditioning the optimization problem for optimal convergence rates of ADMM are discussed. In general, the proposed method can equally be applied with other (first-order) distributed dual algorithms, such as fast/accelerated ADMM [34], [33] or dual (accelerated) gradient algorithms [26], [31].

2) *Theoretical properties:* The following theorem summarizes the closed-loop properties of the proposed inexact DEMPC.

**Theorem 3.** ([24, Thm. 8, Prop. 18]) *Assume that the load  $d(t)$  and the nonlinearity  $c(t)$  are constant<sup>8</sup>. Suppose that the unknown optimal steady-state (8)  $x^e$  lies strictly in the constraint set and that  $(F, A, B)$  is stabilizable. Then, for any set of feasible initial conditions, there exists a sufficiently large  $N$  and small enough prediction error  $\mathcal{W}_{\text{prediction}}$ , such that the MPC problem (10) is recursively feasible for the closed-loop system and the constraints are satisfied for all  $t \geq 0$ . Furthermore, the optimal steady-state  $x^e$  is practically asymptotically stable under the closed-loop dynamics.*

The theoretical guarantees depend on repeated feasibility under inexact minimization. This property is guaranteed by virtue of the proposed formulation in combination with the constraint tightening (17) and the stopping condition (14). The posed assumptions on the power network are mainly stabilizability and strict feasibility of  $x^e$ . The stability properties depend on the prediction horizon  $N$ , for which theoretical bounds can be computed, compare [24, Thm. 8]). These bounds can, however, be conservative. In practice, the prediction horizon  $N$  can be increased in the simulation scenarios till the closed-loop system shows sufficient stability properties. Similarly, the tolerance  $\epsilon$  can be increased till the available computational power is sufficient to satisfy the stopping condition (14) in real-time.

**Remark 4.** *The proposed inexact DEMPC provides bounds on the transient economic performance [19, Sec. 4.4.2], similar to nominal economic MPC [21, Thm. 10]. The closed-loop performance of the proposed inexact DEMPC approaches the infinite horizon optimal performance as the prediction horizon  $N$  increases and the suboptimality of the distributed optimization decreases (small  $\epsilon$ ).*

3) *Multi-Step MPC:* If the computational burden or the communication demand of the proposed method are too large

<sup>7</sup>In principle, a good initialization is given by the candidate solution used in the stability proof [24, Prop. 18]. The computation of this candidate solution can however be quite complex and is not practical for the considered setup with an unknown optimal steady-state.

<sup>8</sup>The properties are equally valid if  $d(t)$  and  $c(t)$  are changing online, if there exists a known bound on the change which is included in the construction of the constraint tightening, compare [35]. Crucially, the construction of the set  $\mathcal{W}_{\text{model}}$  and the constraint tightening are done offline.

for real time operation with the chosen step size  $h$ , using Multi-Step MPC can be quite advantageous. In Multi-Step MPC the first  $M$  parts of the optimized input trajectory are applied and the optimization problem is only solved again after  $M$  time steps. By using Multi-Step MPC, the relaxation  $w_k$  and the corresponding bound  $\mathcal{W}_{\text{inexact}}$  used in the MPC formulation (10) reduce by a factor of  $1/M$  to

$$w_k := \frac{N-1-k}{M}v, \quad \mathcal{W}_{\text{inexact}} = \left\{ w \in \mathbb{R}^n \mid |w| \leq \frac{N}{M}v \right\}.$$

Intuitively, the inexact minimization can be viewed as a potential disturbance. By re-optimizing less frequently the effect of this disturbance is smaller (compare [24, Sec. A.5]). Correspondingly, the computational burden and communication demand can be significantly reduced.

A disadvantage of using Multi-Step MPC is that the controller reacts slower to load fluctuations. This issue is considerably reduced if the MPC is built on top of an existing primary frequency controller (compare Sec. II-A4). Furthermore, for technical reasons<sup>9</sup> it is crucial that  $M$  is not chosen too large (e.g.  $M \leq N/2$ ).

#### IV. SIMULATION STUDY

We show the practicality of the proposed approach by considering various simulation scenarios with the IEEE 39 bus system, which can be seen in Figure 2. We compare the performance of the proposed MPC with unified control (UC) [7]. UC is a continuous time primal-dual algorithm based on a linear network model that asymptotically stabilizes the optimal steady-state (8). The considered simulation scenarios include large load steps, random load fluctuations, and consider economic performance, frequency regulation and constraint satisfaction. **The magnitude of the load changes are chosen such that UC yields significant constraint violations, while the proposed MPC solution ensures safe operation. The large load change can be interpreted as a generator outage and the random load fluctuations reflect standard continuous operation.**

##### A. Simulation setup

The control parameters for UC and the simulation model are taken from [7] with minor modifications<sup>10</sup>. For the DEMPC, we consider a step size of  $h = 0.1s$  and a prediction horizon of  $N = 20$  (equal to 2 seconds). The chosen control parameters<sup>11</sup> are  $\beta = 2 \cdot 10^{-2}$ ,  $\gamma = 10^{-4}$ ,  $\epsilon = 10^{-4}$ , and  $\rho = 0.1$ . The frequency constraints are  $\omega_{\max} = -\omega_{\min} = 0.36$  Hz and  $\delta_{\max} = -\delta_{\min} = 0.4$  rad. The following simulations were obtained using the nonlinear continuous time model, while the

<sup>9</sup>The solution to a finite-horizon economic optimal control problem (such as (9), (10)) has a turnpike property with respect to the desired mode of operation, in the sense that the first part of the trajectory converges to the desired steady-state and the end piece typically diverges (has a leaving arc). Thus, as long as the first  $M$  inputs that are applied to the system do not belong to the second part of the trajectory, the theoretical properties remain valid.

<sup>10</sup>To ensure realistic simulation for the loads, a damping  $D_{\mathcal{L}}$  equal to the smallest generator damping is added.

<sup>11</sup>To improve the numerical conditioning of the online optimization and thus the convergence,  $\delta_i$ ,  $P_i^C$  are scaled by  $10^2$  and  $10^{-3}$ , respectively.

DEMPC controller design uses the affine discrete time model. **A simple hyperbox bound for the model mismatch ( $\mathcal{W}_{\text{model}}$ ) is computed based on simulation experiments<sup>12</sup>.** In the following figures, the power is always relative with respect to the initial condition (previous optimal steady-state).

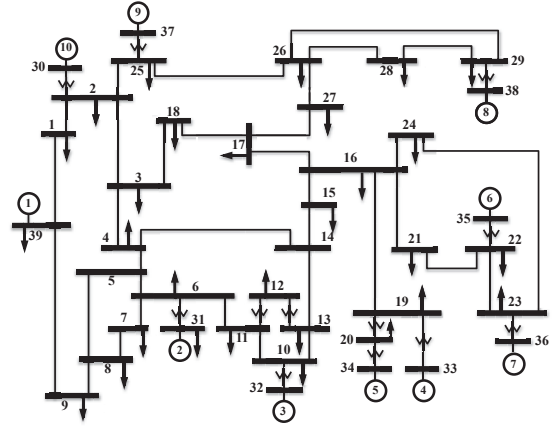


Fig. 2. IEEE 39 bus system: New England from [7]. The network consists of  $|\mathcal{G}| = 10$  generators,  $|\mathcal{L}| = 29$  loads and  $|\mathcal{E}|=46$  lines.

##### B. Economic performance and frequency regulation

We first investigate the economic performance and frequency regulation in two scenarios. For this evaluation, power constraints are not considered.

1) *Large load change:* We first consider a sudden load change of  $P_{30}^L = 1$  pu, which should be economically compensated by increasing the overall power generation. The resulting frequency deviation and power generation for the DEMPC and UC can be seen in Figure 3. As expected based on our theoretical results, both schemes asymptotically converge (close) to the same optimal power generation pattern. The main difference is in the transient response.

The DEMPC achieves fast frequency synchronization, while ensuring the hard bounds on the frequency deviation. The constraint on  $\delta$  (7) ensures that the nominal frequency is recovered. By designing this constraint appropriately, the slower frequency regulation can be tuned independently of the short term frequency synchronization. **This can be thought of as the MPC analog to the integral based control action of AGC.**

UC works with with a continuous time integral based controller, leading to a smooth but slower power generation. Correspondingly, the frequency is regulated slower and larger frequency deviations occur, which violate the frequency constraint. A small frequency offset ( $2 \cdot 10^{-3}$  Hz) persists in the UC, despite the integral based control. The reason is that the controller model in [7] does not incorporate the nonlinear directed power loss, which account for 2 % of the overall power injection in this example. The proposed DEMPC captures the nonlinearity with the affine term  $c(t)$  and ensures zero-offset due to the constraint (7).

<sup>12</sup>In principle, this bound can be analytically derived based on the considered model simplification (e.g. time discretization and linearization). Estimating a bound based on simulation tests is practical and avoids unnecessary conservatism. If the closed-loop is operated smoothly (which can be tuned with  $\gamma$ ), then the magnitude of the model mismatch is smaller.

TABLE I  
QUANTITATIVE COMPARISON - ECONOMIC PERFORMANCE AND FREQUENCY REGULATION

Scenario	Method	$av[\alpha]$	$av[\omega^2]$	$\ \omega\ _\infty$
1) Large load change	UC	0.0127	0.0270	0.502
	DEMPC	0.0138	0.0138	0.336
2) Random fluctuations	UC	0.095	1.223	1.610
	DEMPC	0.328	0.019	0.256

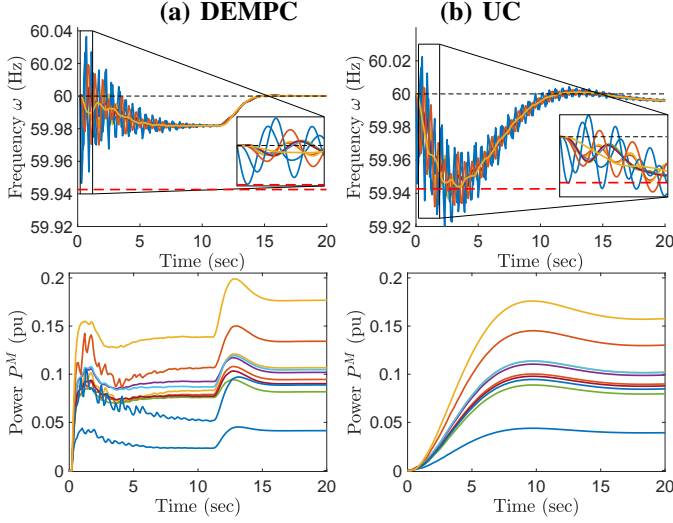


Fig. 3. Large load change: frequency  $\omega_i$  and power generation  $P_i^M$  for all generators (a) controlled by DEMPC or (b) controlled by UC.

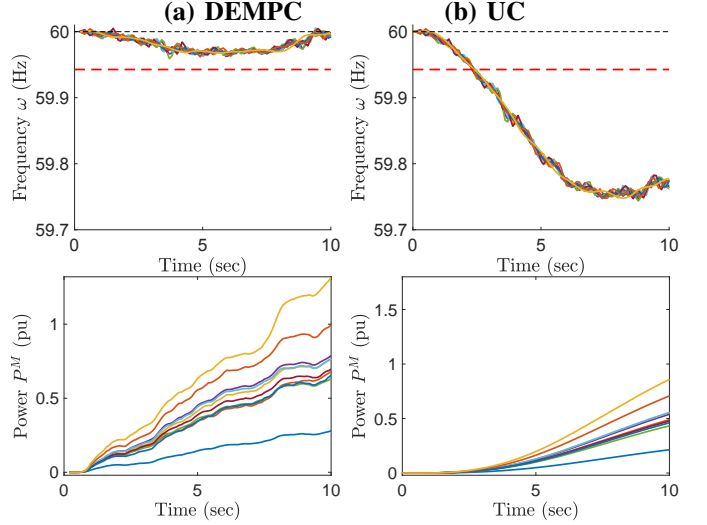


Fig. 4. Random load fluctuations: frequency  $\omega_i$  and power generation  $P_i^M$  for all generators (a) controlled by DEMPC or (b) controlled by UC.

2) *Random fluctuations*: Now we consider fluctuating loads, which we describe with the following stochastic model

$$d_i(k+1) = 0.995 \cdot d_i(k) + \text{unif}(-0.0225, 0.0275), \quad i \in \mathcal{N},$$

where  $\text{unif}$  denotes the uniform distribution. This model results in randomly varying loads, which are bounded and have a positive trend. This kind of scenario represents standard continuous operation and is used for the transient performance evaluation of the controllers. Looking at the resulting trajectories in Figure 4, we can see that both controllers operate with a synchronized frequency. The UC has a relatively slow and smooth power generation, while the DEMPC reacts faster and achieves a significantly smaller frequency deviation. In particular, the DEMPC ensures that the posed frequency limit ( $\omega_{\min}$ ) is satisfied, while the UC shows over four times the allowed frequency deviation. Even though the load  $d$  keeps unpredictably changing, the DEMPC has regulated the frequency deviation close to 0 after only 10s. **The frequency recovery is due to the integral like action, which can be tuned using (7).**

3) *Quantitative comparison*: A quantitative comparison is displayed in Table I. Here  $av[\alpha]$  refers to the average economic

cost of generating power, i.e.,

$$av[\alpha] = \frac{1}{T} \sum_{t=0}^{T-1} \sum_{i \in \mathcal{N}} \alpha_i(P_i^M(t)),$$

where  $T$  is the simulation end time. The average quadratic frequency deviation is  $av[\omega^2]$  and  $\|\omega\|_\infty$  is the maximal frequency deviation. In both scenarios the DEMPC reacts faster, leading to a short-time increase in the power generation (i.e., higher cost  $av[\alpha]$ ) and ensuring significantly faster/better frequency regulation, while satisfying the posed constraints.

### C. Constraint satisfaction

Now we study the ability of the controllers to satisfy hard constraints on the line power flow  $P_{ij}$ . We consider a load change of  $P_{30}^L = 1$  pu and assume that predictions of this load change are available 1s before the change. We consider a line power flow constraint of 0.25 pu in the lines (1, 2); (2, 3); (2, 25), which distribute the load to the network.

The resulting line power flow can be seen in Figure 5. Due to its predictive nature<sup>13</sup>, the DEMPC can react before a constraint violation occurs and successfully satisfy the hard constraint at all time during transient operation. Due to the conservative nature of the proposed constraint tightening, the

<sup>13</sup>The predictive capability of the MPC allows us to incorporate knowledge about future changes in the load, if available. Most other control schemes, like UC, cannot easily take advantage of such knowledge.

closed-loop MPC always keeps a certain distance to the constraint limit. This is inevitable to ensure hard constraint satisfaction despite possible model mismatch and load changes. If the power command  $P_i^C$  is based on a nominal MPC (9) (instead of the proposed formulation (10)), the closed loop will violate the line power constraints instead of keeping a safe distance.

The UC has large constraint violations with fast and strong oscillations in the line power flow. The posed constraint is only satisfied asymptotically after 100s, when the system is in stationary operation (as one might expect based on the theoretical results [7]).

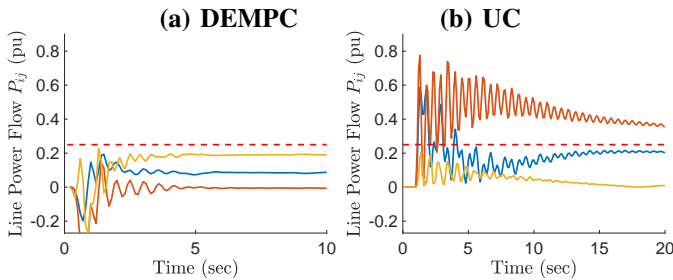


Fig. 5. Large load change: line power flow  $P_{ij}$  (solid) and corresponding limit (dashed) for the lines (1, 2), (2, 3), (2, 25) with (a) DEMPC or (b) UC.

In all the considered simulation scenarios we can see advantages of using MPC. Most notably, the MPC achieves faster frequency stabilization and ensures hard constraint satisfaction (for both frequency deviation  $\omega$  and line power flow  $P_{ij}$ ).

**Remark 5.** A challenge in using MPC is the computational demand of the online optimization. The current implementation using Matlab quadprog on an Intel Core i7 can require up to 65 s of computational time for each time-step. Decreasing the online computation is part of ongoing work, based on a combination of multi-step MPC and incorporation of existing primary frequency controllers. Preliminary results suggest a reduction of the computational demand by a factor of 100 (which is close to real-time), but requires further research. A tailored real-time C implementation (such as [36]) is expected to reduce the computational time again by a factor of 10–100.

## V. CONCLUSIONS

We have proposed an inexact DEMPC scheme to solve the real time economic dispatch problem. In contrast to existing solutions, the proposed controller guarantees stability, constraint satisfaction and economic optimality during transient operation. These theoretical advantages have been confirmed and solidified by the simulation scenarios, in which we achieve fast frequency stabilization and hard constraint satisfaction, despite load fluctuations. We believe that the proposed approach offers a reliable framework to handle increasing fluctuation in future power networks. Future work includes the development of fast MPC schemes to control low inertia power grids.

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